How to Determine the Type of Conic Equation

1. Determine the highest power (degree) of each of the two variables.

2. If one variable has degree 2, while the other has degree 1, the conic is a **parabola**.

   Examples: $8y = x^2$ (parabola that opens up)
   $4x + y^2 = 0$ (parabola that opens left)

3. If both variables are of degree 2, but have different signs (+ and –), the conic is a **hyperbola**.

   Examples: $\frac{x^2}{4} - \frac{y^2}{16} = 1$ (hyperbola aligned parallel to the x-axis)
   $y^2 + 2y - x^2 + 4x = 4$ (hyperbola aligned parallel to the y-axis)

4. If both variables are of degree 2, and have the same signs (+ and + , or – and –), and the coefficients of both squared terms are the same, the conic is a **circle**.

   Examples: $\frac{x^2}{4} + \frac{y^2}{4} = 1$ [circle with center (0,0)]
   $y^2 + 2y + x^2 + 4x = 11$ (circle aligned parallel to the y-axis)

5. If both variables are of degree 2, and have the same signs (+ and + , or – and –), and the coefficients of both squared terms are not the same, the conic is a **ellipse**.

   Examples: $\frac{x^2}{4} + \frac{y^2}{16} = 1$ (ellipse aligned parallel to the y-axis)
   $y^2 + 2y + 2x^2 + 8x = 7$ (ellipse aligned parallel to the y-axis)

6. General case: $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$. Compute the discriminant = $B^2 - 4AC$.
   - $B^2 - 4AC > 0$ → **hyperbola**
   - $B^2 - 4AC = 0$ → **parabola**
   - $B^2 - 4AC < 0$ and $A = C$ and $B = 0$ → **circle**
   - $B^2 - 4AC < 0$ and $A \neq C$ → **ellipse**

Example: $2x^2 + y^2 + 8x + 2y = 7$; A = 2, B = 0, C = 1 → $B^2 - 4AC = -8$ and $A \neq C$ → ellipse

More Examples – What are they?

- $x^2 + 4 - y = 0$
- $x^2 + 2x + 4y^2 = 0$
- $x^2 + 4 - y^2 = 0$
- $x^2 + 2x + y^2 = 0$