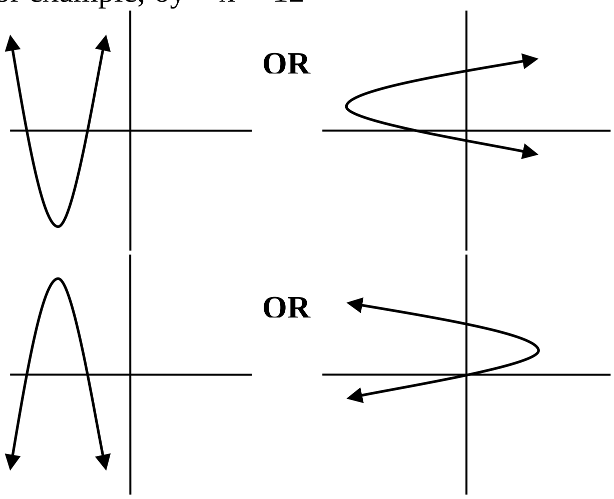
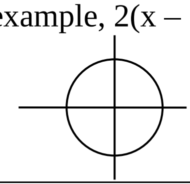
<table>
<tr><th>If You See…</th><th>You Should Think…</th></tr>
<tr><td>A polynomial expression with two variables: with one variable of degree 1 and the other variable of degree 2

For example, $6y + x^2 = 12$
</td>
<td>This is a parabola. Put the equation in one of the two forms:

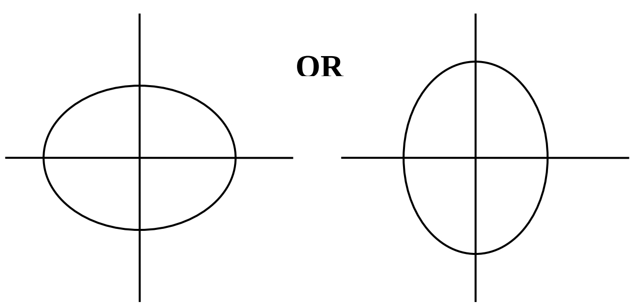
$4p(y – k) = (x – h)^2$ (<u>opens up</u>, if p is positive or <u>opens down</u> if p is negative)
<u>or</u>
$4p(x – h) = (y – h)^2$ (<u>opens right</u>, if p is positive or <u>opens left</u> if p is negative)

Here, p is the distance from the vertex (h, k) to the focus (inside the concave part of the parabola) and in the opposite direction to the directrix line. The directrix line has equation y = k–p for the first case above and x = h–p for the 2^{nd} case.

In the example to the left, rearrange values to give $6(y – 2) = x^2$, so that p = 6/4 = 3/2 and the vertex is at (0, 2). The focus is at (0, 2 + 3/2) = (0, 7/2). The directrix has equation y = 2 – 3/2 or y = ½.</td></tr>
<tr><td>A polynomial expression with two variables: with both variables of degree 2 and with equal coefficients for each 2^{nd}-degree variable.

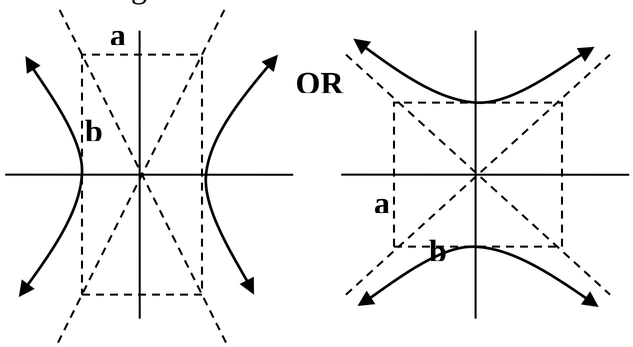
For example, $2(x – 1)^2 + 2(y + 1)^2 = 8$
</td>
<td>This is a circle. The equation for the circle is in the form: $(x – h)^2 + (y – k)^2 = r^2$, where (h, k) is the center of the circle, and r is the radius. If the circle equation has 1^{st}-degree terms (x or y), then complete the square to put in standard form.

In the example to the left, first divide each side of the equation by 2. Then, the center is at (1, –1) and the radius is 2.</td></tr>
<tr><td>A polynomial expression with two variables: with both variables of degree 2 and with different coefficients (of the <u>same</u> sign) for each 2^{nd}-degree variable.

For example, $(x – 1)^2 + \frac{25}{16}(y + 1)^2 = 25$
</td>
<td>This is an ellipse. The equation for the ellipse is in the form: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$, where (h, k) is the center of the ellipse, and a and b are the lengths of half of the two axes. The larger of a or b defines the major axis. The smaller of a or b defines the minor axis. The two foci are at an equal distance c from the center, where (if a > b), $c^2 + b^2 = a^2$ or (if a < b) $c^2 + a^2 = b^2$. If the ellipse equation has 1^{st}-degree terms (x or y), then complete the square to put in standard form.

In the example, first divide each side of the equation by 25. Then, the center is at (1, –1), a = 5, b = 4, and c = 3. The foci are at (–2, –1) and (4, –1). The major axis is parallel to the x-axis (the larger value a^2 divides the $(x – h)^2$ term).</td></tr>
</table>

A **polynomial expression with two variables**: with **both variables of degree 2** and with **different coefficients (of <u>different</u> signs)** for each 2nd-degree variable.



The **asymptotes** are $y = \frac{b}{a}x$ and $y = -\frac{b}{a}x$ (left) or $y = \frac{a}{b}x$ and $y = -\frac{a}{b}x$ (right).

For example, $(x - 1)^2 - \frac{9}{16}(y + 1)^2 = 9$

This is a **hyperbola**. It has two **branches** (curves). The equation for the hyperbola is either: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ or $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$, where (h, k) is the **origin** (center) of the hyperbola, 2a is the distance between the **vertices** and 2b is the distance between the **co-vertices**. The two **foci** are at an equal distance c from the center, where $c^2 = a^2 + b^2$. If the hyperbola equation has 1st-degree terms (x or y), then complete the square to put in standard form. If the $(x - h)^2$ term comes first, the hyperbola opens to the right and left. If the $(y - k)^2$ term comes first, the hyperbola opens up and down.

In the example equation to the left, first divide each side of the equation by 9. Then, the origin is at (1, –1), a = 3, b = 4, and c =5. The foci are at (–4, –1) and (6, –1). The hyperbola opens parallel to the x-axis (opens right and left). The asymptotes are $y = \frac{4}{3}x$ and $y = -\frac{4}{3}x$.