

## Exponential Equations

- There are three types of exponential equations that you'll be given: "exponent = exponent", "exponent = numeric value", and "complex exponential equations".
- You will generally not find an extraneous solution for an exponential equation if you get a real-number solution (because the domain of an exponential function is all real numbers).

### "Exponent = Exponent" Problems

- Solve these easily by equating the exponent values. Convert to the same base first if required.
- Example:
  - $2^{x-4} = 4^{1-x}$ 
    - 1) First, get these into the same base by equating  $4 = 2^2$ . That gives  $2^{x-4} = (2^2)^{1-x} \rightarrow 2^{x-4} = 2^{2-2x}$
    - 2) Just equate the exponents:  $x - 4 = 2 - 2x$
    - 3) Solve for x:  $x = 2$

### "Exponent = Numeric" Value Problems

- Solve these by converting into a logarithmic equation by taking a logarithm on each side of the equal sign.
- You can use any base for the logarithm. However, many times a specific base will help simplify the equation. If using your calculator, base 10 (log) and base e (ln) are the only functions directly available.
- Example:
  - $e^{x-1} = 4$ 
    - 1) Here the base is e, the exponent is  $x - 1$ , and the number is 4. We'll use the natural logarithm (base e).
    - 2) Convert to a logarithmic equation:  $\ln(e^{x-1}) = \ln(4)$ .
    - 3) Using the logarithmic rule for powers, this becomes:  
 $(x - 1)\ln(e) = \ln(4) \rightarrow x - 1 = \ln(4) \rightarrow x = \ln(4) + 1$ .

## Complex Exponential Equation Problems

- Solve these by combining any separate exponential pieces (using the rules of exponents) to get the exponential part by itself, and then using one of the methods above.
- Example:
  - $3(2^{x+1})(2^{2x-2}) - 5 = 7$ 
    - 1) First step: notice that there are two exponents multiplied together. The rules of exponents allow us to combine these into a single exponent – adding the exponents.
    - 2) The equation becomes:  $3(2^{3x-1}) - 5 = 7$ .
    - 3) Now, get the exponential part by itself. Start by adding 5 to each side:  $3(2^{3x-1}) = 12$ .
    - 4) Now divide by 3 to isolate the exponential:  $2^{3x-1} = 4$ .
    - 5) Now, solve by a method above. For example, convert  $4 = 2^2$  to get  $2^{3x-1} = 2^2$ . This gives  $3x - 1 = 2$  or  $x = 1$ .