Factoring Quadratic Polynomials

- What is a quadratic polynomial?
  - The expression is a polynomial: exponents can only be positive integers; the highest power (exponent) is 2 (a square term).
  - Example: $x^2 - 2x + 1$
  - Goal: attempt to convert the polynomial into the <u>product</u> of two simpler expressions.
- Steps:
  1. Factor out a **common factor**
     - The common factor can be a constant (number) or a variable.
     - Example: $2x^2 - 4x + 2 = 2(x^2 - 2x + 1)$ (factor out a 2)
     - Example: $2x^2 - 4x = 2x(x - 2)$ (factor out 2x)
     - Always factor out a negative: $-2x^2 - 3x + 2 = -(2x^2 + 3x - 2)$
  2. Look for a simple special case (**difference of two squares**)
     - Difference of two squares: $p^2 - q^2 = (p + q)(p - q)$. p and q can be any multiple of constants and variables.
     - Example: $x^2 - 4 = (x + 2)(x - 2)$ ($p = x$, $q = 2$)
     - Example: $4x^2 - 25 = (2x + 5)(2x - 5)$ ($p = 2x$, $q = 5$)
     - Note: the **sum of squares** (e.g., x2 + 1) is **not factorable**.
  3. Look for a simple special case (**perfect square trinomial**)
     - Perfect square: $(p + q)^2$ or $(p - q)^2$. p and q can be any multiple of constants and variables.
     - Example: $x^2 - 4x + 4 = (x - 2)(x - 2)$ ($p = x$, $q = 2$)
  4. Factor
     - <u>Simple</u> (using "anti-FOIL"): using example $x^2 + 2x - 3$.
       - Write two parentheses pairs: (   ) (   )
       - The F (first) terms in each parenthesis, when multiplied, give the x^2 term. So, pick x and x: (<u>x</u>   ) (<u>x</u>   )
       - The L (last) term in each parentheses, when multiplied, give the number term (–3). So, find the possible factors of –3; those factors are –3 and 1 or –1 and 3.
       - Which of these two adds to the middle term coefficient (+2)? 3 and –1. Put those two number in the other slots: (x <u>+ 3</u>) (x <u>– 1</u>). Done.

- <u>Split (grouped) middle term</u>: using example $2x^2 – x – 3$.
  - Note: the split term approach is often used when the number in front of x^2 is not 1.
  - In this case, the number in front of x^2 is 2 and the last number is –3. Multiply them together to get –6.
  - Generate all of the factors of –6: $3 \bullet -2, -3 \bullet 2, 1 \bullet -6$, and $-1 \bullet 6$.
  - Determine the sum of each pair of factors: 3 + –2 = 1, 2 + –3 = –1, 1 + –6 = –5, and –1 + 6 = 5.
  - Pick the pair that adds to the number in front of x. In this case the number in front of x is –1, so the 2nd pair of factors, 2 and –3, is the correct pair.
  - Split the middle term into two parts: –x = 2x + –3x, using the pair of factors just selected. Then, the original equation $2x^2 – x – 3 = 2x^2 + 2x + –3x – 3$.
  - Split this into two groups: $2x^2 + 2x + –3x – 3 = (2x^2 + 2x) + (–3x – 3)$.
  - Factor out any common factor in each piece: $(2x^2 + 2x) + (–3x – 1) = 2x(x + 1) – 3 (x + 1)$.
  - Note that there is a common factor in each term: (x + 1)
  - Factor out the common factor (x + 1) to get the final factoring: (2x – 3) (x + 1).