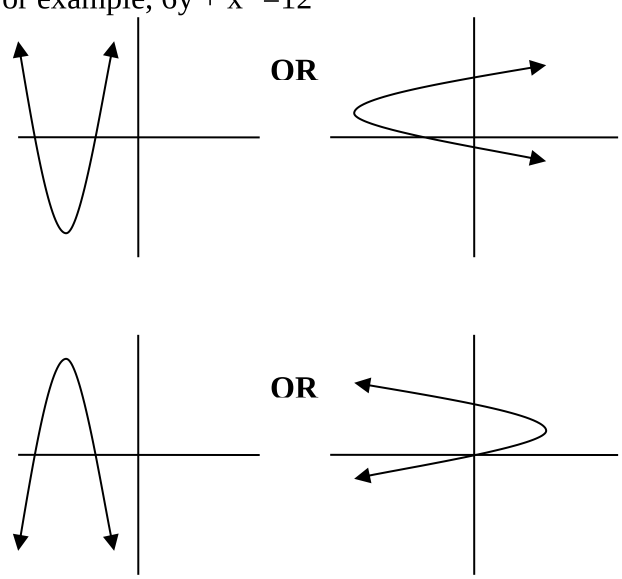
If You See…	You Should Think…
A **polynomial function**: For example, $f(x) = 2x^3 + 3x^2 – 4$	The **domain** of a polynomial is all real numbers. The **range** of a polynomial depends on the function – if not obvious, then graph it.
A **quadratic equation:** For example, $x^2 + 2x + 3 = 0$	Either factor or use the quadratic formula to get solutions. For a 2nd-degree equation (quadratic) there are no more than 2 solutions (sometimes only one distinct solution and sometime none). In this example, the equation factors in to $(x + 2)(x + 1) = 0$. Then, the key point: any time two numbers a and b are multiplied and give 0 ($ab = 0$), the either $a = 0$, $b = 0$, or both. In this case this gives two solutions: $x = –1$ and $x = –2$.
A **general polynomial function with constraints:** For example, $P(x) = Ax^2 + Bx + C$ of degree 2 with two roots/zeroes at –1 and ½ and $P(0) = –1$	Use the constraints to determine A, B, and C. Use $P(0) = –1$ to determine C. Use the two roots/zeroes to determine A and B. In this example, $P(0) = A(0)2 + B(0) + C = –1$ gives $C = –1$. The two roots/zeroes means that $P(–1) = 0$ and $P(½) = 0$. For the first case, this gives $P(–1) = A(–1)^2 + B(–1) + C = 0$, which yields the equation $A – B – 1 = 0$. The other root/zero gives $A/4 + B/2 – 1 = 0$. Solving these two simultaneous equations gives $A = 2$ and $B = 1$. Therefore, $P(x) = 2x^2 + x – 1$.
A **polynomial expression with two variables**: with **one variable of degree 1 and the other variable of degree 2** For example, $6y + x^2 =12$ 	This is a **parabola**. Put the equation in one of the two forms: $4p(y – k) = (x – h)^2$ (opens up, if p is positive or opens down if p is negative) or $4p(x – h) = (y – h)^2$ (opens right, if p is positive or opens left if p is negative) Here, p is the distance from the **vertex** (h, k) to the **focus** (inside the concave part of the parabola) and in the opposite direction to the **directrix line**. The directrix line has equation $y = k–p$ for the first case above and $x = h–p$ for the 2nd case. In the example to the left, rearrange values to give $6(y – 2) = x^2$, so that $p = 6/4 = 3/2$ and the vertex is at (0, 2). The focus is at $(0, 2 + 3/2) = (0, 7/2)$. The directrix has equation $y = 2 – 3/2$ or $y = ½$.

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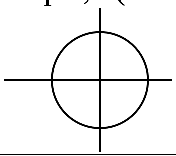
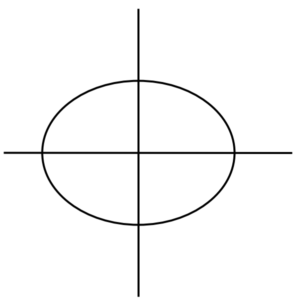
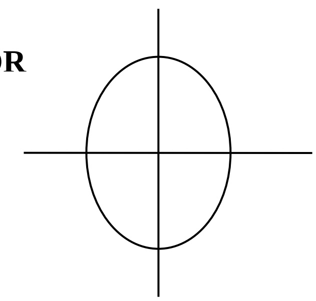
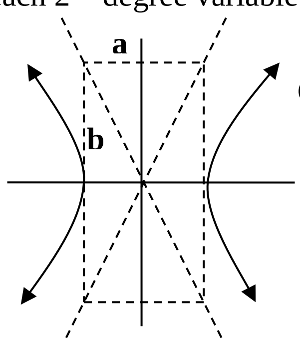
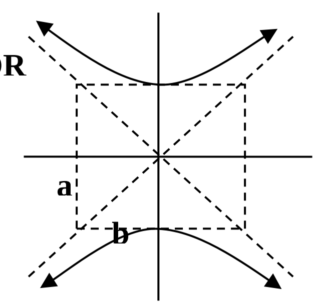
If You See…	You Should Think…
A **polynomial expression with two variables**: with **both variables of degree 2** and with **equal coefficients** for each 2^nd^-degree variable. For example, $2(x-1)^2 + 2(y+1)^2 = 8$ 	This is a **circle**. The equation for the circle is in the form: $(x-h)^2 + (y-k)^2 = r^2$, where (h, k) is the **center** of the circle, and r is the **radius**. If the circle equation has 1st-degree terms (x or y), then complete the square to put in standard form. In the example to the left, first divide each side of the equation by 2. Then, the center is at (1, –1) and the radius is 2.
A **polynomial expression with two variables**: with **both variables of degree 2** and with **different coefficients (of the <u>same</u> sign)** for each 2nd-degree variable. For example, $(x-1)^2 + \frac{25}{16}(y+1)^2 = 25$  **OR** 	This is an **ellipse**. The equation for the ellipse is in the form: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$, where (h, k) is the **center** of the ellipse, and a and b are the lengths of half of the two axes. The larger of a or b defines the **major axis**. The smaller of a or b defines the **minor axis**. The two **foci** are at an equal distance c from the center, where (if a > b), $c^2 + b^2 = a^2$ or (if a < b) $c^2 + a^2 = b^2$. If the ellipse equation has 1st-degree terms (x or y), then complete the square to put in standard form. In the example, first divide each side of the equation by 25. Then, the center is at (1, –1), a = 5, b = 4, and c = 3. The foci are at (–2, –1) and (4, –1). The major axis is parallel to the x-axis (the larger value a^2 divides the $(x-h)^2$ term).
A **polynomial expression with two variables**: with **both variables of degree 2** and with **different coefficients (of <u>different</u> signs)** for each 2nd-degree variable.  **OR**  The **asymptotes** are $y = \frac{b}{a}x$ and $y = -\frac{b}{a}x$ (left) or $y = \frac{a}{b}x$ and $y = -\frac{a}{b}x$ (right). **For example**, $(x-1)^2 - \frac{9}{16}(y+1)^2 = 9$	This is a **hyperbola**. It has two **branches** (curves). The equation for the hyperbola is either: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ or $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$, where (h, k) is the **origin** (center) of the hyperbola, 2a is the distance between the **vertices** and 2b is the distance between the **co-vertices**. The two **foci** are at an equal distance c from the center, where $c^2 = a^2 + b^2$. If the hyperbola equation has 1st-degree terms (x or y), then complete the square to put in standard form. If the $(x-h)^2$ term comes first, the hyperbola opens to the right and left. If the $(y-k)^2$ term comes first, the hyperbola opens up and down. In the example equation to the left, first divide each side of the equation by 9. Then, the origin is at (1, –1), a = 3, b = 4, and c =5. The foci are at (–4, –1) and (6, –1). The hyperbola opens parallel to the x-axis (opens right and left). The asymptotes are $y = \frac{4}{3}x$ and $y = -\frac{4}{3}x$.

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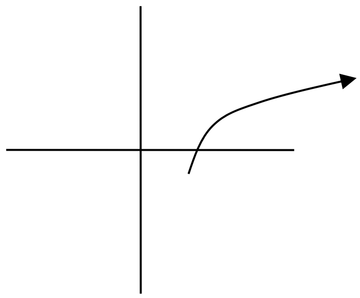
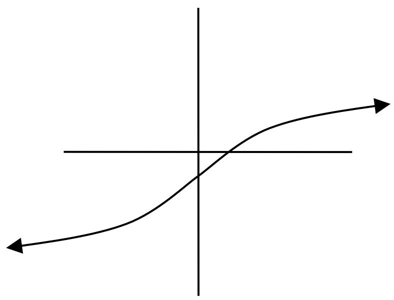
If You See…	You Should Think…
A **radical function** with an **even** root (½, ¼, etc.): For example, f(x) = $\sqrt{x+3}-1$ 	The **domain** of a radical function with an <u>even root</u> is all real numbers such that the **radicand** (the part under the radical sign) is ≥ 0. The **range** of the square root function y = $\sqrt{x}$ is all real numbers ≥ 0. Any number added to or subtracted from the square root changes the range. In this example, x + 3 ≥ 0 implies that x ≥ -3. These values of x make up the **domain**. The range of f(x) in this example is all real numbers ≥ -1.
A **radical function** with an **odd** root (1/3, 1/5, etc.): For example, f(x) = $\sqrt[3]{x+3}-1$ 	The **domain** of a radical function with an odd root is all real numbers. The **range** is all real numbers.
A **radical expression** with a radical sign: For example, $\sqrt{18}$	**Simplify** as much as possible. In this example, $\sqrt{18} = \sqrt{9*2} = \sqrt{9}*\sqrt{2} = 3\sqrt{2}$
A **radical expression** with a fractional exponent: For example, $8^{\frac{2}{3}}$	**Simplify** as much as possible. Usually, first do the operation (usually the <u>root</u>) that makes the problem simpler. In this example, $8^{\frac{2}{3}} = \left[(8)^{\frac{1}{3}}\right]^2 = [2]^2 = 4$ Note: the exponent applies only to the item to its immediate left.
A **radical equation**: For example, $2\sqrt{2-x} = x-2$	A standard technique is to raise each side of the equation to the same power, removing the radical sign. You must check for **extraneous** (invalid) solutions. In this example, squaring each side gives 4(2 – x) = $(x – 2)^2$. Solving, gives x = 2 and – 2. Plugging these back into the original equation shows that x = – 2 is not a valid solution (extraneous). So, the only solution is x = 2.

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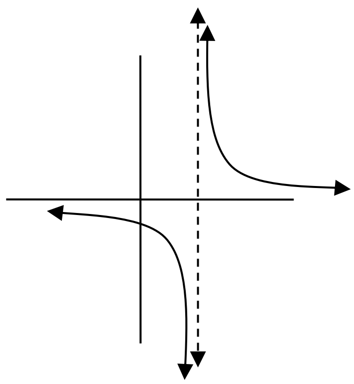
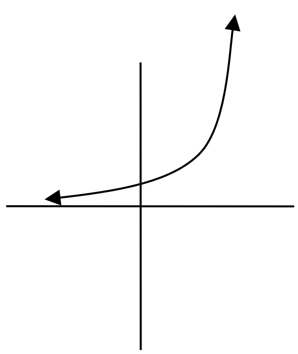
If You See…	You Should Think…
A **rational function** (with an variable in the denominator): For example, f(x) = $\frac{1}{x-1}$ 	The **domain** of a rational function is all real numbers such that the **denominator** is $\neq 0$. A value of x where the denominator is 0 is called an **excluded value**. The **range** of a rational function depends on the function – if not obvious, then graph it. **Vertical asymptotes** may occur at each **excluded value**. The asymptotes are equations of lines of the form x = a, where a is one of the excluded values. In this example, there is one vertical asymptote with equation x = 1. The **horizontal asymptote** is the equation of a line toward which the function approaches for very large x and very small (large negative) x. In this example, the horizontal asymptote is the x-axis, y = 0.
A **rational expression**: For example, $\frac{x-1}{x+1}+\frac{2}{x+1}$	Realize that any rational expression works just like a **fraction**: you add, subtract, multiply, divide, and simplify them using the same basic rules. In the example, there is already a least common denominator, so add the numerators and simplify: $= \frac{(x-1)+2}{x+1} = \frac{x+1}{x+1} = 1$
An **exponential function**: For example, f(x) = e^{2x} or g(x) = $2^{-x}+1$  The function **f(x) = b·a^{x}** is a **growth function** if a > 1. It is a **decay function** if a < 1. The function **f(x) = b·a^{-x}** is a **growth function** if a < 1. It is a **decay function** if a > 1.	The **domain** of an exponential function is all real numbers. The **range** of an exponential function is generally all real numbers ≥ 0, but depends on the function (where constants are added or subtracted from the exponential part) – if not obvious, then graph it. The **horizontal asymptote** is the equation of a line toward which the function approaches for very small (large negative) x (for positive exponents) or very large x (for negative exponents). In this example, the horizontal asymptote for f(x) is the x-axis, y = 0, while the horizontal asymptote for g(x) is y = 1 (the exponential part goes to 0 as x gets very large). Examples: f(x) = 2^{x} is a **growth function**. g(x) = 2^{-x} is a **decay function**. h(x) = $\left(\frac{1}{2}\right)^{x}$ is a **decay function**.

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If You See…	You Should Think…
An **exponential expression**: For example: $2^2 * 2^3 = 2^{2+3} = 2^5$ $\frac{2^5}{2^2} = 2^{5-2} = 2^3$ $(2^2)^3 = 2^{2*3} = 2^6$ $(\frac{5}{4})^3 = \frac{5^3}{4^3}$ $5^{-2} = \frac{1}{5^2}$ $2^{\frac{2}{3}} = \sqrt[3]{2^2}$	You must remember each of the **exponent rules** shown in the examples to the left. You can at least remember how to use a simple example to recreate the rule if you forget. For example, for the first rule, put in an intermediate step: $2^2 * 2^3 = (2*2)*(2*2*2) = 2^5$; so, when you multiply two exponent expressions with the same base, then you add the exponents.
An **exponential equation**: For example: $2^{x-3} = 4^{x+1}$	A standard technique is to transform all bases to use a standard base, using the standard exponential rules. For example, in the example at left, change 4 into 2^2. That results in an equation $2^{x-3} = 2^{2x+2}$. Because the same base (2) is raised to two different exponents but gives the same result, the two exponents must be equal. So, x – 3 = 2x + 2, which results in x = –5.
A **special exponential function**: **Growth function**: $N = N_0 e^{rt}$, where N is the population at time t, N_0 is the initial population at t = 0, r = the rate of growth (positive r) or decline (negative r), and t is the time (in whatever units are needed for the problem) **Compound interest function (continuous)**: $A = Pe^{rt}$, where A is the final principal (amount of money), P is the initial principal, r is the interest rate, and t is the time. **Compound interest function (periodic)**: $A = P(1-\frac{r}{n})^{nt}$, where A, P, r, and t are as above, and n is the number of compoundings per year.	Each of these functions is in general form. The task is usually to find the particular (specific) function, where you find the value of r. For the **growth function**, you are usually given the initial population N_0 at t = 0 and the final population N at some later time $t = t_1$. You plug in those values to find out the specific value of r. For the **compound interest function**, you are usually given the initial principal P and the rate r, and are asked to find the final principal at some time $t = t_1$. You plug in those values to find A. For periodic compound interest, you'll also be told n (e.g., if there is annual compounding, n = 1; if there is monthly compounding, n = 12; if there is quarterly compounding, n = 4).

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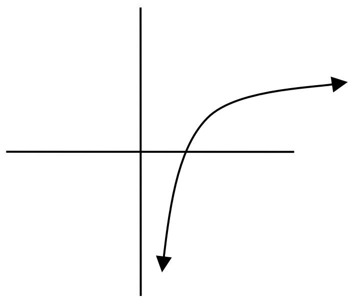
If You See…	You Should Think…
A **logarithmic function**: For example, f(x) = log(x-1) 	The logarithm function operates only on real numbers ≥ 0. Therefore, the **domain** of a logarithmic function is all real numbers such that the number operated on by the logarithm is ≥ 0. The **range** of a logarithmic function is all real numbers. In the example to the left, the domain is ≥ 1, because the value operated on by the logarithm is x – 1. You can set x – 1 ≥ 0 and solve for x to get the domain.
A **logarithmic expression**: For example: $\log(2 \bullet 3) = \log 2 + \log 3$ $\log(\frac{2}{3}) = \log 2 - \log 3$ $\log(2^3) = 3\log 2$	You must remember each of the **logarithm rules** shown in the examples to the left. Remember, a logarithm works just like an exponent. These expressions match the corresponding rules for exponents. For example, when you multiply two numbers, you add their exponents. Similarly, the logarithm of two numbers multiplied together = the logarithm of each number added together.
Logarithmic simplification examples: 3log x – 2 log y + 4 log z = $\log\left(\frac{x^3z^4}{y^2}\right)$ $\log\left(x^2yz^3\right)$ = 2log x + log y + 3log z	These are some typical examples for simplifying logarithmic expressions.
A **logarithmic equation**: For example: $\log_4 2^{x-3} = x+1$ **OR** log (2x) = log (3 – x)	A standard technique is to transform the logarithm equation into an equivalent exponential equation (if you have a logarithm = a number). If you have a logarithm = a logarithm, then simply equate the numbers on which the logarithms operate. You must check for **extraneous** (invalid) solutions. For example, in the first example at left, the definition of a logarithm gives $2^{x-3} = 4^{x+1}$. This gives the same result for x as the example under "exponential equation" (x = –5). Plugging this back into the original equation shows that this is a valid solution (no extraneous solutions). In the second example at left, just equate 2x = 3 – x and solve for x. That gives x = 1, which is valid (no extraneous solutions).

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If You See…	**You Should Think…**
A **sequence**: For example: 2, 6, 10, 14, 18, …	Check the difference between each element. If it is <u>constant</u>, it is an **arithmetic sequence**. You get each successive term by <u>adding</u> the constant to the previous term. If a_1 is the first term, a_2 is the second term, …, a_n is the nth term, and d is the constant difference between terms, then the **recursive rule** is $a_n = a_{n-1} + d$. The **explicit rule** (in terms of the first term, a_1) is $a_n = a_1 + (n-1)d$. If you add up each of the terms in the sequence, you get the **arithmetic series** ($a_1 + a_2 + \ldots + a_n$). The **sum** of the first n terms of the series = $S_n = n\frac{(a_1 + a_n)}{2}$. In the example, the constant difference is 4 (2 + 4 = 6; 6 + 4 = 10; etc.).
A **sequence**: For example: 2, 6, 18, 54, 162, …	Check the difference between each element. If there is a **constant ratio between each element**, it is a **geometric sequence**. You get each successive term by <u>multiplying</u> the constant ratio by the previous term. If a_1 is the first term, a_2 is the second term, …, a_n is the nth term, and r is the constant ratio between terms, then the **recursive rule** is $a_n = ra_{n-1}$. The **explicit rule** (in terms of the first term, a_1) is $a_n = r^{n-1}a_1$. If you add up each of the terms in the sequence, you get the **geometric series** ($a_1 + a_2 + \ldots + a_n$). The **sum** of the first n terms of the series = $S_n = a_1\frac{(1 - r^n)}{1 - r}$. In the example, the constant ratio is 3 (2 * 3 = 6; 6 * 3 = 18; etc.).

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