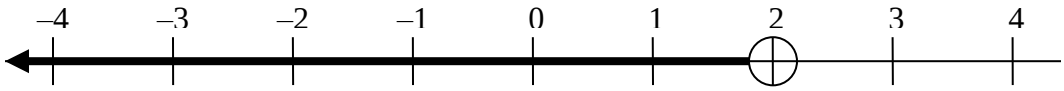
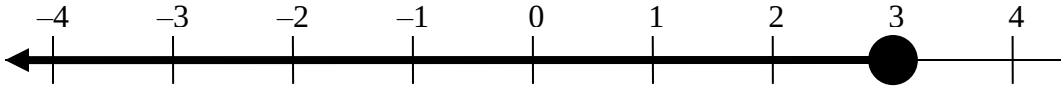
Inequalities

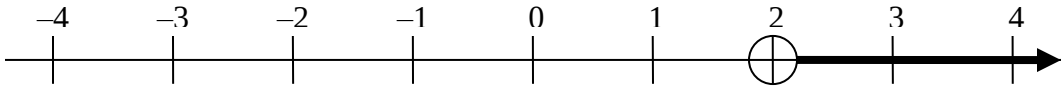
- **Inequality symbols**: < means "less than" (small side of < is to the left); > means "greater than" (large side of > is to the left).
  - A given number is "less than" another number if it has a smaller value than the other number
    - Examples: 1 is less than 2. 4 is less than 8.
    - These can be written as $1 < 2$ and $4 < 8$.
    - You can also represent <u>all</u> numbers x that are less than 2 as **x < 2**.
  - A given number is "greater than" another number if it has a larger value than the other number
    - Examples: 3 is greater than 2. 4 is greater than –1.
    - These can be written as $3 > 2$ and $4 > -1$.
    - You can also represent <u>all</u> numbers x that are greater than 2 as **x > 2**.
  - A given number is "less than or equal to" another numbers if it has a smaller value than the other number <u>or</u> is equal to the number.
    - Examples: 1 is less than or equal to 3. 1 is less than or equal to 1.
    - These can be written as $1 \leq 3$ and $1 \leq 1$.
    - You can also represent <u>all</u> numbers x that are less than or equal to 3 as **x ≤ 3**.
  - A given number is "greater than or equal to" another numbers if it has a larger value than the other number <u>or</u> is equal to the number.
    - Examples: 4 is greater than or equal to 3. –1 is greater than or equal to –1.
    - These can be written as $4 \geq 2$ and $-1 \geq -1$.
    - You can also represent <u>all</u> numbers x that are greater than or equal to –1 as **x ≥ –1**.
- Graphing a simple inequality
  - You can use the number line to show the values that are less than or greater than a given number.
    - First, plot the given number. If the inequality is < or >, put an open circle at the number. If the inequality is ≤ or ≥, then put a filled-in circle at the number.
    - Second, draw an arrow in the direction of the inequality: + to right, – to left.
    - Examples:
      - **x < 2**



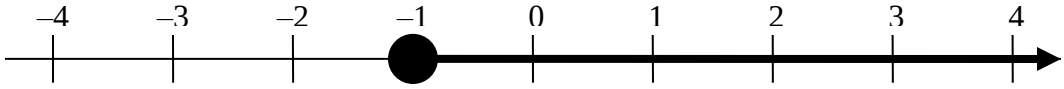
      - **x ≤ 3**



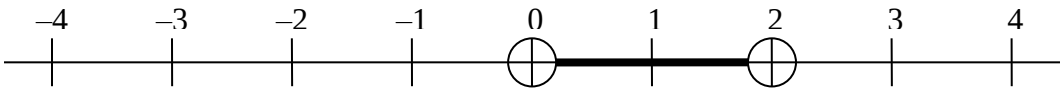
      - **x > 2**



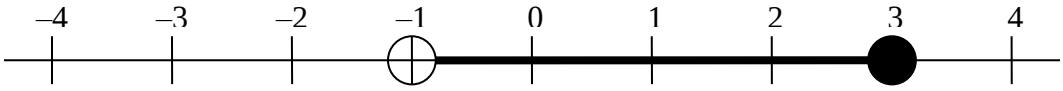
      - **x ≥ –1**



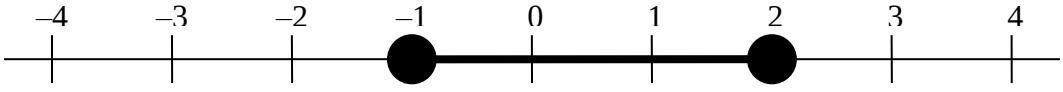
- Compound inequalities ("between")
  - A given number is "between" two other numbers if it is less than the larger **and** greater than the smaller.
    - Examples: 3 is between 2 and 7. 4 is between –1 and 8.
  - You can express the "between" relationship using two separate inequalities combined with "**and**."
    - Example: 3 is between 2 and 7 may be expressed as 3 is greater than 2 and 3 is less than 7.
  - You can express the "between" relationship using < and >.
    - Examples: 3 is between 2 and 7 may be written as $2 < 3 < 7$. This is really two inequalities: $2 < 3$ and $3 < 7$.
  - You can also express the "between" relationship using $\leq$ and $\geq$ to include the "end values."
    - Examples: 3 is between 2 and 7, including 2 and 7, may be written as $2 \leq 3 \leq 7$. 4 is between –1 and 8, including –1, may be written as $-1 \leq 4 < 8$.
  - Moreover, you can describe all of the points between two numbers by using a variable.
    - Examples: all of the numbers x between 2 and 7 may be written as $2 < x < 7$. All of the numbers x between –1 and 8, including 8, may be written as $-1 < x \leq 8$.
- Graphing a "between" inequality
  - You can use the number line to show the values that are between two given numbers.
    - First, plot the given numbers. If the inequality is < or >, put an open circle at the number. If the inequality is $\leq$ or $\geq$, then put a filled-in circle at the number.
    - Then draw a solid line between the two numbers.
    - Examples:
      - **$0 < x < 2$**



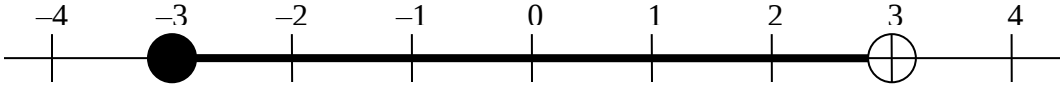
      - **$-1 < x \leq 3$**



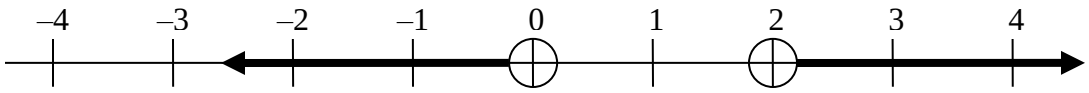
      - **$-1 \leq x \leq 2$**



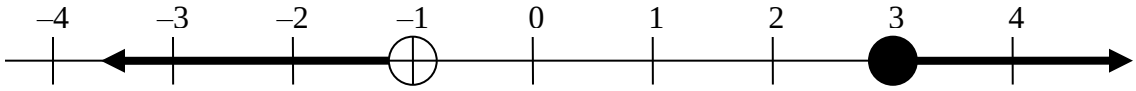
      - **$x \geq -3$ and $x < 3$**



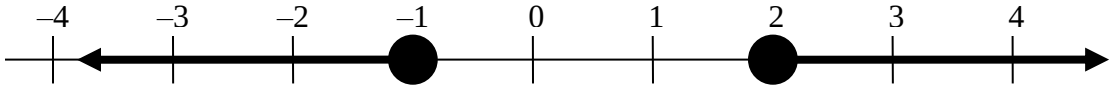
- Compound inequalities ("not between" using an **or** statement)
  - A number is <u>not between</u> 2 other numbers if it is greater than the larger **<u>or</u>** less than the smaller.
    - Examples: 3 is not between –1 and 2 because 3 > 2.
  - You can express the "outside" relationship using a variable and two separate inequalities combined with "<u>or</u>" :
    - Example: x is greater than 2 <u>or</u> x is less than –1 may be expressed as $x > 2$ <u>or</u> $x < -1$.
  - You can also express the "between" relationship using $\leq$ and $\geq$ to include the "end values."
    - Examples: x is outside 2 and 7, but includes 2 and 7, may be written as $x \geq 2$ <u>or</u> $x \leq 7$.
- Graphing a "not between" inequality
  - You can use the number line to show the values that are "not between" two given numbers.
    - First, plot the given numbers. If the inequality is < or >, put an open circle at the number. If the inequality is $\leq$ or $\geq$, then put a filled-in circle at the number.
    - Then draw solid arrows to the left and right of the two numbers.
    - Examples:
      - **$x < 0$ <u>or</u> $x > 2$**



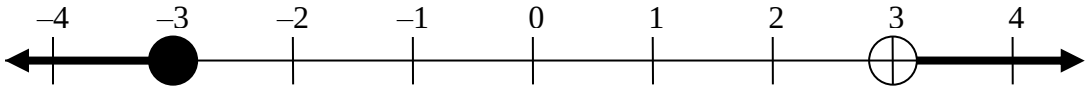
      - **$x < -1$ <u>or</u> $x \geq 3$**



      - **$x \leq -1$ <u>or</u> $x \geq 2$**

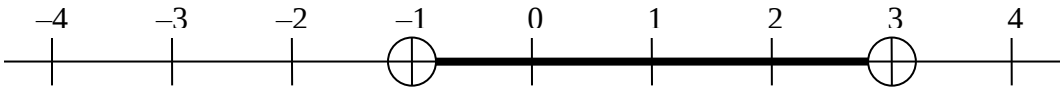


      - **$x \leq -3$ <u>or</u> $x > 3$**

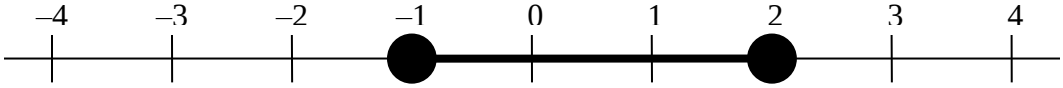


- Absolute value inequalities
  - The absolute value of a number is the value of the number when made positive.
  - If the number is positive, then its absolute value has the same value. If the number is negative, then its absolute value is the negative of the number.
    - Examples: $|3| = 3$ and $|-3| = 3$. $|0| = 0$.
  - On the number line the absolute value of the difference of two numbers represents the distance between the two numbers.
    - Examples: the distance between 3 and 1 is $|3 - 1| = 2$. The distance between 3 and –1 is $|3 - (-1)| = |3 + 1| = 4$
  - You can express an absolute value using a variable.
    - Example: the distance between an unknown point x and the point –1 is $|x - (-1)| = |x + 1|$.
  - You can express the set of values a given distance from a given point using an absolute value inequality.
    - Example: the set of all points that are 3 units from the point 1 is given by $|x - 1| < 3$.

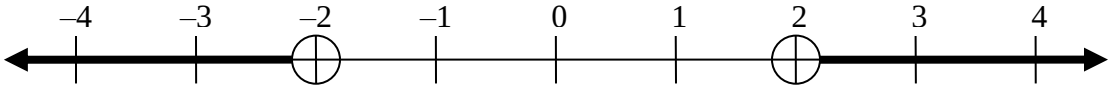
- Graphing an absolute value inequality (less than)
  - You can use an absolute value inequality to describe certain "between" relationships.
    - First, given two endpoints find the midpoint of the two numbers by adding them and dividing by two (the average of the two numbers).
    - Second, find the distance between the two endpoints and divide by two (average).
    - The form of the equation then becomes | x – midpoint | < distance (or ≤ distance).
    - If the inequality is < or >, put an open circle at the number. If the inequality is ≤ or ≥, then put a filled-in circle at the number.
    - Then draw a solid line between the two numbers.
    - Examples:
      - **Endpoints –1 and 3 → midpoint = (–1 + 3)/2 = 1 and distance = | 3 – (–1) | / 2 = 2**
      - **The absolute value form of the inequality is | x – 1 | < 2 (same as –1 < x < 3).**



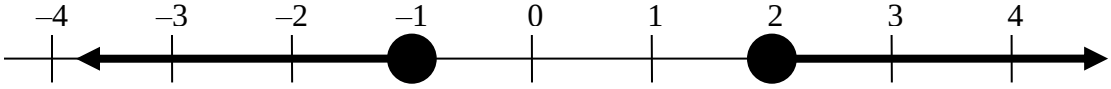
      - **Endpoints –1 and 2 → midpoint = (–1 + 2)/2 = ½ and distance = | 2 – (–1) | = 3**
      - **The absolute value form of the inequality is | x – ½ | < 3 (same as –1 ≤ x ≤ 2).**



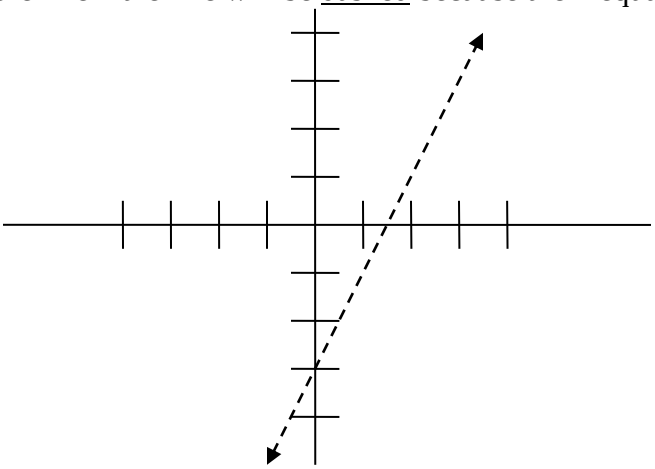
- Graphing an absolute value inequality (greater than)
  - You can use an absolute value inequality to describe certain "not between" relationships.
    - First, given two endpoints find the midpoint of the two numbers by adding them and dividing by two (the average of the two numbers).
    - Second, find the distance between the two endpoints and divide by two (average).
    - The form of the equation then becomes | x – midpoint | > distance (or ≥ distance).
    - If the inequality is >, put an open circle at the number. If the inequality is ≥, then put a filled-in circle at the number.
    - Then draw solid arrows outside the two endpoints.
    - Examples:
      - **Endpoints –2 and 2 →** midpoint = (–2 + 2)/2 = 0 and distance = | 2 – (–2) |/ 2 = 2
      - The absolute value form of the inequality is **| x – 0 | > 2** (same as x > 2 <u>or</u> x < –2).



      - **Endpoints –1 and 2 →** midpoint = (–1 + 2)/2 = ½ and distance = | 2 – (–1) |/ 2 = 3/2
      - The absolute value form of the inequality is **| x – ½ | ≥ 2** (same as x ≥ 2 <u>or</u> x ≤ –1).



- Two-variable inequalities (linear equation)
  - The "edge" of the inequality area will be a line. If the equation includes < or >, the line will be dashed. If the equation includes $\leq$ or $\geq$ the line will be solid.
  - The inequality area will be on one side of the line or the other – not both.
  - Steps:
    1. Replace the <, >, $\leq$, or $\geq$ with an = sign to find the equation of the "edge" line.
    2. Graph the "edge" line – either dashed or solid, as described above.
    3. Pick a point (coordinate) on one side of the line. Substitute that coordinate into the original inequality.
       - If the inequality remains true, then shade all points on the same side of the line as the point you picked.
       - If the inequality becomes false, then shade all points on the opposite side of the line from the point you picked.
  - Example – $y < 2x - 3$:
    1. Get the equation of the "edge" line: $y = 2x - 3$.
    2. Graph the line – the line will be dashed because the inequality is < (not $\leq$).



    3. Pick an easy point on one side of the line. I'll choose (0,0), because that's easy to plug in. That gives $(0) < 2(0) - 3$ or $0 < -3$. That is not a true statement, so I'll shade all of the region on the other side of the line: