Logarithmic Equations

- There are three types of logarithmic equations that you'll be given: "logarithm = logarithm", "logarithm = numeric value", and "complex logarithmic equations".
- Always check for "extraneous solutions" by substituting solutions back into the original equation.

"Logarithm = Logarithm" Problems

- Solve these easily by equating the logarithm values.
- Example:
  - log (2x + 1) = log (4 – x)
    1) Just equate the values: 2x + 1 = 4 – x
    2) Solve for x: x = 1
    3) Substitute in original: log (2·1 + 1) = log (4 – 1) gives log (3) = log (3) → true!

"Logarithm = Numeric Value" Problems

- Solve these by converting into an exponential equation. Remember: $\text{base}^{\text{exponent}}$ = number.
- Example:
  - log (4x + 2) = 1
    1) Here the base is 10, the exponent is 1, and the number is 4x + 2.
    2) Convert to an exponential equation: $10^1 = 4x + 2$ or 10 = 4x + 2.
    3) Solve for x: x = 2.
    4) Substitute in original: log (4·2 + 2) = log (10) = 1 → true!

Complex Logarithmic Equation Problems

- Solve these by combining any separate logarithm pieces (using the rules of logarithms) to get a single logarithmic expression by itself, and then using one of the methods above.

- Example:
  - $\log_3(x + 2) + \log_3(x) = 1$
    1) First step: notice that there are two logarithms added together. The rules of logarithms allow us to combine this into a single logarithm – multiplying the numeric values.
    2) The equation becomes: $\log_3(x + 2)x = 1$ or $\log_3(x^2 + 2x) = 1$.
    3) Here the base is 3, the exponent is 1, and the number is $x^2 + 2x$.
    4) Convert to an exponential equation: $3^1 = x^2 + 2x$ or $x^2 + 2x - 3 = 0$ (quadratic).
    5) Solve for x by factoring: $x = 1, -3$.
    6) Substitute 1 in original: $\log_3(1 + 2) + \log_3(1) = \log_3(3) + \log_3(1) = 1 + 0 = 1$ $\rightarrow$ true!

       Substitute –3 in original: $\log_3(-3 + 2) + \log_3(-3) = \log_3(-1) + \log_3(-3) \neq 1$ $\rightarrow$ false! Because the logarithm cannot operate on a negative number, the solution $x = -3$ is <u>extraneous</u>.

       So, there is only one solution: $x = 1$.

- Example:
  - $\log(x + 4) - \log(5 - x) = \log(1/2)$
    1) First step: notice that there are two logarithms subtracted. The rules of logarithms allow us to combine this into a single logarithm – dividing the numeric values.
    2) The equation becomes: $\log\left(\frac{x+4}{5-x}\right) = \log\left(\frac{1}{2}\right)$.
    3) That gives: $\frac{x+4}{5-x} = \frac{1}{2}$. Cross-multiplying gives $2(x + 4) = 5 - x$. Solve for x to get $x = -1$.
    4) Substitute –1 in original: $\log\left(\frac{-1+4}{5--1}\right) = \log\left(\frac{1}{2}\right)$ $\rightarrow$

       $\log\left(\frac{3}{6}\right) = \log\left(\frac{1}{2}\right)$ $\rightarrow$ true!