If You See…	You Should Think…
A **quadratic expression that is to be <u>factored</u>:** For example, $x^2 + 6x - 16$	If possible, factor into (__ +/– __) (__+/– __). How to do this? Realize that *factoring is the reverse of FOIL*. Namely: • The first terms in each set of parentheses must <u>multiply</u> to equal the x^2 term. • The last two terms must <u>multiply</u> to the equal the number term. So, *find the factors of the number term*. • The factors of the number term must <u>add</u> to equal the coefficient of the middle term. In this example: • The x^2 term factors into x and x. • The last term factors into 1 and –16, or –1 and 16, or 2 and –8, or 8 and –2, or 4 and –4. • The correct pair is 8 and –2, since that <u>adds</u> to 6 (the coefficient of the 6x middle term). • The correct factors are $(x + 8)(x - 2)$.
A **quadratic equation to be solved by <u>factoring</u>:** For example, $x^2 + 6x - 16 = 0$	Factor as described above. In this example: • The equation factors into $(x + 8)(x - 2) = 0$. • *The key point*: any time two numbers **a** and **b** are multiplied and give 0 ($ab = 0$), then either $a = 0$, $b = 0$, or both. • In this case, that says that $x + 8 = 0$ or $x - 2 = 0$. This gives two solutions: $x = -8$ and $x = 2$.

A **quadratic equation to be solved where there are a <u>squared term</u> and <u>numbers</u>:**
 	
For example, $2(x - 1)^2 - 8 = 10$
	Solve as follows:
	• Get the squared term by itself on one side of the equal sign by moving all other number terms to the other side of the equal sign.
	    • Add (to remove a negative term)
	    • Subtract (to remove a positive term)
	    • Multiply (to remove a value dividing the square term)
	    • Divide (to remove a value multiplying the square term).
	• Take the square root of each side of the equation. *Note: that inserts a ± in front of the number*.
	• Finally, solve the equation for x.
	
	• In this example
	    • $2(x - 1)^2 - 8 \mathbf{+ 8} = 10 \mathbf{+ 8} \rightarrow 2(x - 1)^2 = 18$
	    • $2(x - 1)^2 \mathbf{/ 2} = 18 \mathbf{/ 2} \rightarrow (x - 1)^2 = 9$
	    • Take the square root, giving $x - 1 = \pm 3$.
	    • That gives 2 solutions: $x = 4$, $x = -2$.

A **quadratic equation to be solved by completing the square:**
 	
Example 1: $x^2 + 6x – 16 = 0$
 	
Example 2: $2x^2 + 8x – 3 = 0$
	This uses a different approach:
	• Put parentheses around the x^2 and x terms.
	• Factor out any coefficient of x^2 (if there is one, it is called “a”).
	• The coefficient of the middle term is called “b”. Compute b/2 and $(b/2)^2$.
	• Add $(b/2)^2$ inside the parentheses.
	• Multiply $(b/2)^2$ by the factored out coefficient of x^2 and subtract that value outside of the parentheses.
	• Factor the parentheses as $\mathbf{a\ (x + b/2)^2}$.
	• Solve as above: a quadratic equation with a squared term and numbers.
	
	In Example 1:
	• $\mathbf{(}x^2 + 6x \quad \mathbf{)} – 16 = 0$
	    • $b = 6;\ b/2 = 3;\ (b/2)^2 = 9$
	• $(x^2 + 6x \mathbf{+ 9}) \mathbf{– 9} – 16 = 0$
	• $(x + 3)^2 – 25 = 0$
	• Solve: $(x + 3)^2 – 25 = 0 \rightarrow x = 3 \pm 5 = 8, –2$
	In Example 2:
	• $\mathbf{(}2x^2 + 8x \quad \mathbf{)} – 3 = 0$
	• $\mathbf{2}(x^2 + 4x \quad) – 3 = 0$
	    • $a = 2;\ b = 4;\ b/2 = 2;\ (b/2)^2 = 4$
	• $2(x^2 + 4x \mathbf{+ 4}) \mathbf{– 2(4)} – 3 = 0$
	• $2(x + 2)^2 – 11 = 0$
	• Solve: $2(x + 2)^2 – 11 = 0 \rightarrow x = -2 \pm \sqrt{\frac{11}{2}}$

A quadratic function to be put into vertex form: $\mathbf{y = ax^2 + bx + c \rightarrow y = a\ (x – h)^2 + k}$ For example, $y = x^2 + 6x – 16$	Convert to vertex form by completing the square: • Put parentheses around the x^2 and x terms. • Factor out any coefficient of x^2 (if there is one, it is called “a”). • The coefficient of the middle term is called “b”. Compute b/2 and $(b/2)^2$. • Add $(b/2)^2$ inside the parentheses. • Multiply $(b/2)^2$ by the factored out coefficient of x^2 and subtract that value outside of the parentheses. • Factor the parentheses as $\mathbf{a\ (x + b/2)^2}$. • Simplify (by adding any number terms together). The vertex is (h, k), where h = –b/2 In the example: • $y = (x^2 + 6x \quad) – 16$ • $b = 6;\ b/2 = 3;\ (b/2)^2 = 9$ • $y = (x^2 + 6x \mathbf{+ 9}) \mathbf{– 9} – 16$ • $y = (x + 3)^2 – 25 \rightarrow$ vertex: (–3, –25)