**Solving Linear Systems of Equations ("Simultaneous Equations")**

- There are two basic methods you'll learn to solve a system of equations:
  - **Graphing** (for equations that are easy to graph)
  - **Elimination** of one variable (for any equations)

- There are four methods you'll learn to solve a system of equations by **elimination**:
  - **Add** the two equations
  - **Subtract** the two equations
  - Solve one equation for one variable and **substitute** into the other equation
  - **Combination** of the above methods: multiply the first equation by one constant and the second equation by another constant. Then add or subtract as above.

- Steps to perform when **graphing**:
  - Put each equation into the form y = mx + b (slope-intercept form of the line)
  - Check the <u>slope</u> (m) of each line. If the two <u>slopes</u> are <u>equal</u>, then the lines are <u>parallel</u>. In that case, there is **no solution**.
  - Otherwise, carefully plot the two lines. Carefully determine where they cross. The coordinate of that point **(x, y) is the solution** to the original system of equations.
  - Note: this approach is only useful when there are <u>two equations</u> and when the equations are "nice" – i.e., generally the <u>solutions</u> must have <u>integer</u> values.

- Steps to perform when using **elimination**:
  - First, determine if a variable in one equation has its opposite (negative) in the other equation. If so, add the equations to eliminate that variable.
  - If not, then determine if a variable in one equation has its equal in the other equation. If so, subtract the equations to eliminate that variable.
  - If not, see if one equation can be easily written with one variable in terms of the other. If so, then substitute for that variable in the second equation.
  - After finding one variable's value, that value is substituted into one of the original equations. Then the second variable's value may be obtained.
  - In each of these cases, the purpose is to combine the equations into one equation, eliminating one of the variables. The combined equation will then have only one variable and can be easily solved.

**Example of Each System of Equation Solution Method**

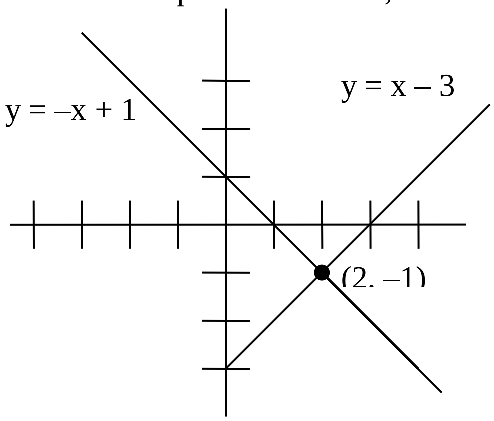
Graphing

System of equations: $x + y = 1$

$x - y = 3$

1. Convert to $y = mx + b$ form:
   $x + y = 1 \rightarrow y = -x + 1$ (slope = –1)
   $x - y = 3 \rightarrow y = x - 3$ (slope = 1)

2. The slopes are different, so carefully graph:




Addition

System of equations: $x + y = 1$
$x - y = 3$

1. Note that y is the opposite of – y. Therefore, add the two equations:

$$\begin{array}{r} x + y = 1 \\ +\,(x - y = 3) \\ \hline 2x \quad = 4 \end{array}$$

2. Solving for x gives $x = 2$. Then substitute that value into one of the two original equations; in this example we'll pick $x + y = 1$: $(2) + y = 1 \rightarrow y = -1$.
3. **Solution is (x, y) = (2, –1).**

Subtraction

System of equations: $x + y = 1$
$x - y = 3$

1. Note that both equations contain x. Therefore, subtract the two equations:

$$\begin{array}{r} x + y = 1 \\ -\,(x - y = 3) \\ \hline 2y = -2 \end{array}$$

2. Solving for y gives $y = -1$. Then substitute that value into one of the two original equations; in this example we'll pick $x + y = 1$: $x + (-1) = 1 \rightarrow x = 2$.
3. **Solution is (x, y) = (2, –1).**

Substitution

System of equations: $x + y = 1$

$x - y = 3$

1. Solve the first equation for x: $x + y = 1 \rightarrow x = 1 - y$
2. Substitute that value for x into the second equation wherever you see x:
   $x - y = 3 \rightarrow (1 - y) - y = 3$
3. Solving for y, that gives $y = -1$. Substitute into either equation to get $x = 2$ as before.

More Complex Example:

System of equations: $3x + 2y = -1$

$2x - 3y = 1$

1. Note that the two y-terms have opposite signs (but different coefficients). We'll plan to set these equations up to allow us to perform addition.
2. Multiply the first equation by 3: $3\ (3x + 2y = -1) \rightarrow 9x + 6y = -3$
3. Multiply the second equation by 2: $2\ (2x - 3y = 1) \rightarrow 4x - 6y = 2$
4. Add the two equations to get $13x = -1 \rightarrow x = -1/13$.
5. Substitute into either equation to get $y = -5/13$.