If You See…	**You Should Think…**
f(x) is a continuous function on [a,b]. Which values must f(x) take?	**Intermediate Value Theorem** If f(x) is continuous on [a,b], it must take all values between f(a) and f(b). That simply means that a continuous function does not skip any values between its start and end points. Note: however, it can also take finite values larger or smaller than f(a) and f(b). There is no limit on f(x) taking finite values that are not between f(a) and f(b).
f(x) is a continuous function on [a,b]. Must f(x) attain both a maximum and a minimum on [a,b]?	**Extreme Value Theorem** If f(x) is continuous on [a,b], it must attain both a maximum and a minimum on that interval.
f(x) is a continuous function on [a,b] and is differentiable on (a,b). Is the average slope $\frac{\Delta y}{\Delta x}$ between the endpoints ever equal to the derivative in the interval?	**Mean Value Theorem for Derivatives** If f(x) is continuous on [a,b] and differentiable on (a,b), there must be at least one point c in the interval where $\frac{\Delta y}{\Delta x} = \frac{f(b)-f(a)}{b-a} = f'(c)$. Note: there may be more than one value c where $\frac{\Delta y}{\Delta x} = f'(c)$.
f(x) is a continuous function on [a,b] and is differentiable on (a,b) and f(a) = f(b) = 0. Must there be a value c in the interval where f'(c) = 0?	**Rolle's Theorem** If f(x) is continuous on [a,b] and differentiable on (a,b) and **f(a) = f(b) = 0**, there must be at least one point c in the interval where f'(c) = 0.
f(x) is differentiable on (a,b)	f(x) differentiable on (a,b) implies **f(x) is continuous** on (a, b).
f(x) is a continuous function on [a,b] and define the anti-derivative function: $F(x) = \int_a^x f(t)dt$. **Is F(x) differentiable and what is its derivative?**	**Fundamental Theorem of Integral Calculus I** $F'(x) = \frac{d}{dx}\int_a^x f(t)dt = f(x)$. Note: this theorem says that any continuous function f(x) has an antiderivative F(x) and the derivative of the anti-derivative is the original function f(x). Note: if the upper bound is a function of x, then the derivative is found using the chain rule: $F'(x) = \frac{d}{dx}\int_a^{g(x)} f(t)dt = f(g(x))g'(x)$.
f(x) is a continuous function on [a,b] and with anti-derivative F(x). What is $\int_a^b f(x)dx$**?**	**Fundamental Theorem of Integral Calculus II** $\int_a^b f(x)dx = F(b) - F(a)$. This means that one may find the definite integral by finding the anti-derivative F(x) and evaluating at the bounds, a and b.
f(x) is a continuous function on [a,b]. What is the average value of f(x) on the interval?	**First Mean Value Theorem for Integrals** $f_{avg} = \frac{1}{b-a}\int_a^b f(x)dx$ This means that the average of f(x) on [a,b] is the area divided by the interval length.