If You See…	You Should Think…
The integral of a power function 1. $\int x^3 dx$ 2. $\int x^{1.5} dx$	**Integration power rule:** $\int x^n dx = \frac{x^{n+1}}{n+1}$ 1. $\int x^3 dx = \frac{x^{3+1}}{3+1} = \frac{x^4}{4}$ 2. $\int x^{1.5} dx = \frac{x^{1.5+1}}{1.5+1} = \frac{x^{2.5}}{2.5}$
The integral of a trig function 1. $\int \sin x\, dx$ 2. $\int \cos x\, dx$ 3. $\int \sec^2 x\, dx$ 4. $\int \sec x \tan x\, dx$ 5. $\int \csc^2 x\, dx$ 6. $\int \csc x \cot x\, dx$	**Simple trig function integration:** 1. $\int \sin x\, dx = -\cos x + C$ 2. $\int \cos x\, dx = \sin x + C$ 3. $\int \sec^2 x\, dx = \tan x + C$ 4. $\int \sec x \tan x\, dx = \sec x + C$ 5. $\int \csc^2 x\, dx = -\cot x + C$ 6. $\int \csc x \cot x\, dx = -\csc x + C$
The integral of exponent functions 1. $\int e^x\, dx$ 2. $\int 2^x\, dx$	**Exponential function integration:** 1. $\int e^x\, dx = e^x + C$ 2. $\int 2^x\, dx = \int \left(e^{\ln 2}\right)^x dx = \int e^{x \ln 2} dx = \frac{e^{x \ln 2}}{\ln 2} + C = \frac{2^x}{\ln 2} + C$
The integral of 1/x $\int \frac{1}{x} dx$	**Exponential function integration:** $\int \frac{1}{x} dx = \ln \mid x \mid + C$
The integral of a quotient (with simple divisor) $\int \frac{x^3 - 2x^2 + 1}{x} dx$	**Simplify by dividing (simple division)** $\int \frac{x^3 - 2x^2 + 1}{x} dx = \int \left(\frac{x^3}{x} - \frac{2x^2}{x} + \frac{1}{x} \right) dx = \int \left(x^2 - 2x + \frac{1}{x} \right) dx$
The integral of a quotient (with complex divisor) $\int \frac{x^3 - 2x^2 + 1}{x+1} dx$	**Simplify by dividing (long division)** $\int \frac{x^3 - 2x^2 + 1}{x+1} dx = \int \left(x^2 - 3x + 3 - \frac{2}{x+1} \right) dx$
The integral of a complex function with derivative also present 1. $\int \frac{3x^2 - 4x}{x^3 - 2x^2 + 1} dx$ 2. $\int \tan x dx = \int \frac{\sin x}{\cos x} dx$ 3. $\int x\sqrt{x^2 + 1} dx$	**Simplify by u-substitution** 1. let u = $x^3 - 2x^2 + 1$; then du = $(3x^2 - 4x)$dx $\int \frac{3x^2 - 4x}{x^3 - 2x^2 + 1} dx = \int \frac{du}{u} = \ln \mid u \mid + C = \ln \mid x^3 - 2x^2 + 1 \mid + C$ 2. let u = $\cos x$; then du = $-\sin x\, dx$ $\int \tan x dx = \int \frac{\sin x}{\cos x} dx = \int \frac{-du}{u} = -\ln \mid u \mid + C = -\ln \mid \cos x \mid + C$ 3. let u = $x^2 + 1$; then du = 2xdx $\int x\sqrt{x^2+1} dx = \int \sqrt{u} \frac{du}{2} = \frac{1}{2} \int u^{\frac{1}{2}} du = \frac{u^{\frac{3}{2}}}{3} + C = \frac{\left(x^2+1\right)^{\frac{3}{2}}}{3} + C$

The integral of a quotient with a squared term in the divisor 1. $\int \frac{1}{\sqrt{1-x^2}}dx$ 2. $\int \frac{-1}{\sqrt{1-x^2}}dx$ 3. $\int \frac{1}{1+x^2}dx$ 4. $\int \frac{1}{x\sqrt{x^2-1}}dx$ 5. $\int \frac{-1}{1+x^2}dx$ 6. $\int \frac{-1}{x\sqrt{x^2-1}}dx$	**Simplify by trig-substitution** 1. let x = $\sin\theta$; then dx = $\cos\theta d\theta$ and $\theta = \sin^{-1}x$ $\int \frac{1}{\sqrt{1-x^2}}dx = \int \frac{\cos\theta d\theta}{\sqrt{1-\sin^2\theta}} = \int \frac{\cos\theta d\theta}{\cos\theta} = \int d\theta = \theta + C = \sin^{-1}x + C$ 2. let x = $\cos\theta$; then dx = $-\sin\theta d\theta$ and $\theta = \cos^{-1}x$ $\int \frac{-1}{\sqrt{1-x^2}}dx = \int \frac{-(-\sin\theta)d\theta}{\sqrt{1-\cos^2\theta}} = \int \frac{\sin\theta d\theta}{\sin\theta} = \int d\theta = \theta + C = \cos^{-1}x + C$ 3. let x = $\tan\theta$; then dx = $\sec^2\theta d\theta$ and $\theta = \tan^{-1}x$ $\int \frac{1}{1+x^2}dx = \int \frac{\sec^2\theta d\theta}{1+\tan^2\theta} = \int \frac{\sec^2\theta d\theta}{\sec^2\theta} = \int d\theta = \theta + C = \tan^{-1}x + C$ 4. let x = $\sec\theta$; then dx = $\sec\theta\tan\theta d\theta$ and $\theta = \sec^{-1}x$ $\int \frac{1}{x\sqrt{x^2-1}}dx = \int \frac{\sec\theta\tan\theta d\theta}{\sec\theta\sqrt{\sec^2\theta-1}} = \int \frac{\sec\theta\tan\theta d\theta}{\sec\theta\tan\theta} =$ $\int d\theta = \theta + C = \sec^{-1}\lvert x\rvert + C$ 5. let x = $\cot\theta$; then dx = $-\csc^2\theta d\theta$ and $\theta = \cot^{-1}x$ $\int \frac{-1}{1+x^2}dx = \int \frac{-\csc^2\theta d\theta}{1+\cot^2\theta} = \int \frac{-(-\csc^2\theta)d\theta}{\csc^2\theta} = \int d\theta = \theta + C = \cot^{-1}x + C$ 6. let x = $\csc\theta$; then dx = $-\csc\theta\cot\theta d\theta$ and $\theta = \csc^{-1}x$ $\int \frac{-1}{x\sqrt{x^2-1}}dx = \int \frac{-(-\csc\theta\cot\theta)d\theta}{\csc\theta\sqrt{\csc^2\theta-1}} = \int \frac{\csc\theta\cot\theta d\theta}{\csc\theta\cot\theta} =$ $\int d\theta = \theta + C = \csc^{-1}\lvert x\rvert + C$
Integration of a complex product $\int x e^x dx$	**Integration by parts:** $\int u dv = uv - \int v du$ let u = x and dv= $e^x dx \Rightarrow du = dx \text{ and } v = e^x$ $\int x e^x dx = x e^x - \int dx = x e^x - x + C$
Integration of a complex fraction $\int \frac{1}{x^2-2x-3}dx$	**Integration by partial fractions:** $\int \frac{1}{x^2-2x-3}dx = \int \frac{\frac{1}{4}}{x-3}dx + \int \frac{-\frac{1}{4}}{x+1}dx = \frac{1}{4}\ln\lvert x-3\rvert - \frac{1}{4}\ln\lvert x+1\rvert + C$ $= \frac{1}{4}\ln\left\lvert \frac{x-3}{x+1}\right\rvert + C$