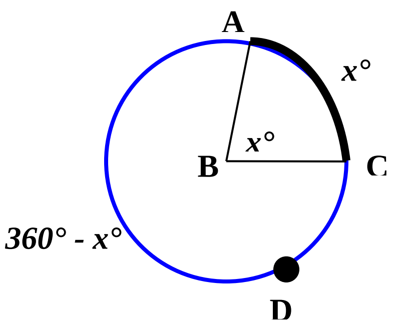
Circles – Central Angle, Arcs

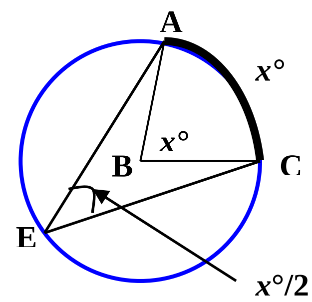


Minor Arc:

$$m\overset{\frown}{AC} = m\angle ABC = x^\circ$$

Major Arc:

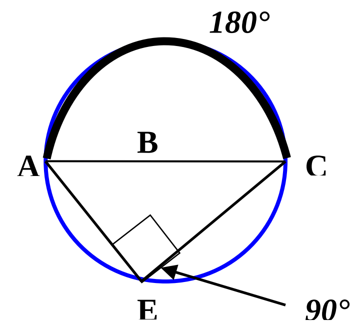
$$m\overset{\frown}{ADC} = 360^\circ - x^\circ$$



$\angle AEC$ is an inscribed angle. Its measure is ½ of the central angle $\angle ABC$.

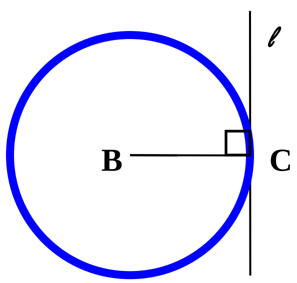
$$\angle AEC = \frac{1}{2}\angle ABC$$

Compare with "Secant, Chord, and Tangent Angles" below – all have factor ½.

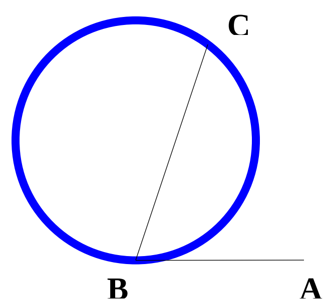


If the central angle intercepts a diameter (180°), then the inscribed angle is 90°.

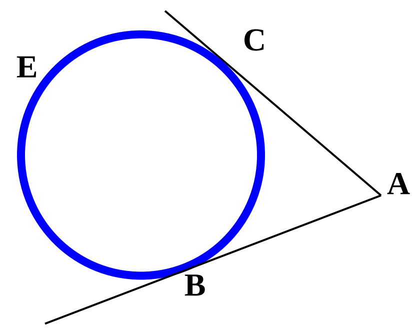
Circles – Tangents



Tangent line ℓ is perpendicular to radius BC at point C.

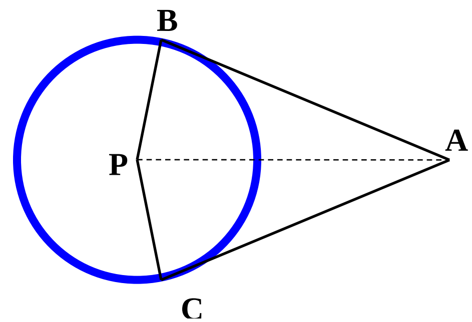


For a tangent $\overline{AB}$ and a secant $\overline{BC}$ drawn from a common point B, the $m\angle CBA = \frac{1}{2} m\overset{\frown}{CB}$.



If two tangents intersect <u>outside</u> a circle, then the angle formed is <u>half the difference</u> of the intercepted arcs.

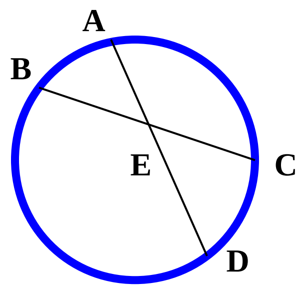
$$m\angle CAB = \frac{1}{2}(m\overset{\frown}{CEB} - m\overset{\frown}{CB})$$



For two tangents $\overline{AB}$ and $\overline{AC}$ drawn from a common point A, the lengths of $\overline{AB}$ and $\overline{AC}$ are the same. Indeed, the triangles ΔABP and ΔACP are congruent.

Compare with "Secant, Chord, and Tangent Angles" below – all have a factor of ½.

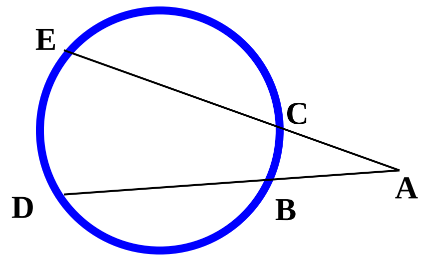
Circles – Secant, Chord, and Tangent Angles



If two secants or chords intersect inside a circle, then each angle formed is half the sum of the intercepted arcs.

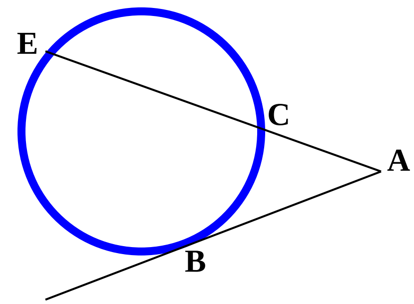
Example:

$$m\angle BED = \frac{1}{2}(m\overset{\frown}{BD} + m\overset{\frown}{AC})$$



If two secants intersect outside a circle, then the angle formed is half the difference of the intercepted arcs.

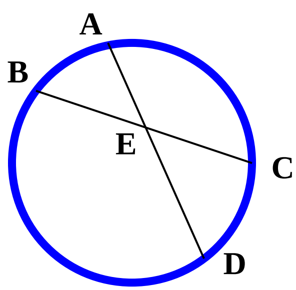
$$m\angle CAB = \frac{1}{2}(m\overset{\frown}{ED} - m\overset{\frown}{CB})$$



If a secant and a tangent intersect outside a circle, then the angle formed is half the difference of the intercepted arcs.

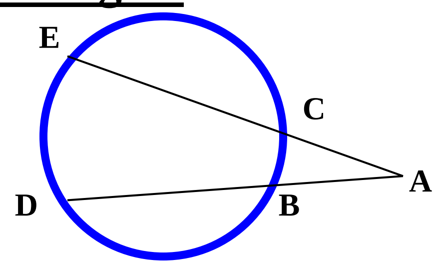
$$m\angle CAB = \frac{1}{2}(m\overset{\frown}{EB} - m\overset{\frown}{CB})$$

Circles – Secant and Chord Lengths



If two chords intersect inside a circle, then the products of the lengths of the two pieces of each chord are equal.

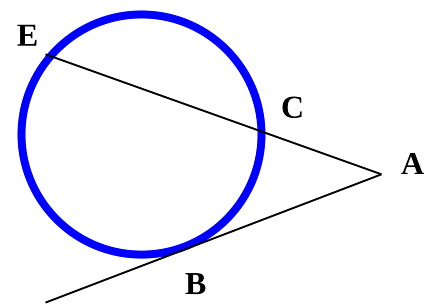
$$AE * ED = BE * EC$$



If two secants intersect outside a circle, then the products of the full segment lengths and the external segment lengths are equal.

$$AE * AC = AD * AB$$

OR



If a secant and a tangent intersect outside a circle, then the product of the full secant segment length and the external segment length equals the square of the tangent segment length.

$$AE * AC = (AB)^2$$