Limit Analysis

Limits at a Finite Value of x (x → a)

Type 1 (continuous function that exists at x = a):

- Approach: plug in the value of x =a to evaluate f(a).
- Example: $\lim_{x \to 3}(2x-4) = 2(3)-4 = 2$
- Example: $\lim_{x \to 3}\frac{(2x-4)}{x^2-5} = \frac{2(3)-4}{4} = \frac{1}{2}$

Type 2 (rational function that evaluates to $\frac{0}{0}$ at x = a):

- Approach: Factor the rational function. If the factor (x – a) exists in the numerator and denominator, cancel the common factors and plug in x = a to evaluate the remaining function.
- Example: $\lim_{x \to 5}\frac{x^2-3x-10}{x-5} = \lim_{x \to 5}\frac{(x-5)(x+2)}{x-5} = \lim_{x \to 5}(x+2) = 5+2 = 7$

Limits at an Infinite Value of x (x →± ∞)

Type 1 (simple rational function):

- Approach: A positive power of ***x*** in the denominator forces the limit to 0. A positive of ***x*** in the numerator forces the limit to 0.
- Example: $\lim_{x \to \infty}\frac{1}{x^r} = 0$ if r > 0.
- Example: $\lim_{x \to \infty} x^r = \infty$ if r > 0.

Type 2 (rational function with simple denominator):

- Approach: divide the numerator terms individually by the denominator and take the limit of each piece separately.
- Example: $\lim_{x \to \infty}\frac{2x+3}{3x} = \lim_{x \to \infty}\frac{2x}{3x} + \lim_{x \to \infty}\frac{3}{3x} = \lim_{x \to \infty}\frac{2}{3} + \lim_{x \to \infty}\frac{1}{x} = \frac{2}{3} + 0 = \frac{2}{3}$

Type 3 (rational function with complex denominator):

- Approach: divide the numerator terms individually by the highest power in the denominator. Then evaluate the limit of the numerator and denominator separately.
- Example: $\lim_{x \to \infty}\frac{2x+3}{3x^2-5} = \lim_{x \to \infty}\frac{\frac{2x}{x^2}+\frac{3}{x^2}}{\frac{3x^2}{x^2}-\frac{5}{x^2}} = \frac{\lim_{x \to \infty}\left(\frac{2x}{x^2}+\frac{3}{x^2}\right)}{\lim_{x \to \infty}\left(\frac{3x^2}{x^2}-\frac{5}{x^2}\right)} = \frac{0+0}{3-0} = \frac{0}{3} = 0$

Type 4 (rational function with positive exponential term):

- Approach: The exponential term approaches ∞ much faster than any power of n. If the exponential term is in the numerator, the limit is forced to → ∞. If the exponential term is in the denominator, the limit is forced to 0.
- Example: $\lim_{x\to\infty} \frac{2^x}{3x} = \infty$
- Example: $\lim_{x\to\infty} \frac{3x}{2^x} = 0$

Type 5 (rational function with negative exponential term):

- Approach: $(-1)^n$ oscillates between –1 and 1 as n → ∞. If the limit of the remaining terms in the function → 0, then the oscillating function → 0. Otherwise the limit does not exist (is undefined).
- Example: $\lim_{x\to\infty} \frac{(-1)^x}{3x} = \frac{\lim_{x\to\infty}(-1)^x}{\lim_{x\to\infty} 3x} = \lim_{x\to\infty}(-1)^x \bullet \lim_{x\to\infty} \frac{1}{3x} = \pm 1 \bullet 0 = 0$
- Example: $\lim_{x\to\infty} \frac{(-1)^n(2n+3)}{3n} = \lim_{x\to\infty}(-1)^n \bullet \lim_{x\to\infty} \frac{(2n+3)}{3n} = \pm 1 \bullet \frac{2}{3} = \textit{undefined}$