

Polynomial Analysis - Example

Given: a polynomial $f(x) = x^3 - x^2 - 4x + 4$

Step 1:

- The degree of the polynomial is 3. There will be 3 roots.

Step 2:

- The leading coefficient is +1.
- Because the degree is odd and the leading coefficient is > 0 , the graph decreases to the left and increases to the right (N shape).

Step 3:

- The candidate rational roots are: $\frac{4}{1}$, $\frac{2}{1}$, $\frac{1}{1}$, $\frac{-4}{1}$, $\frac{-2}{1}$, and $\frac{-1}{1}$
 - Pick a candidate root (pick a simple one first – say, +1) and perform synthetic division:

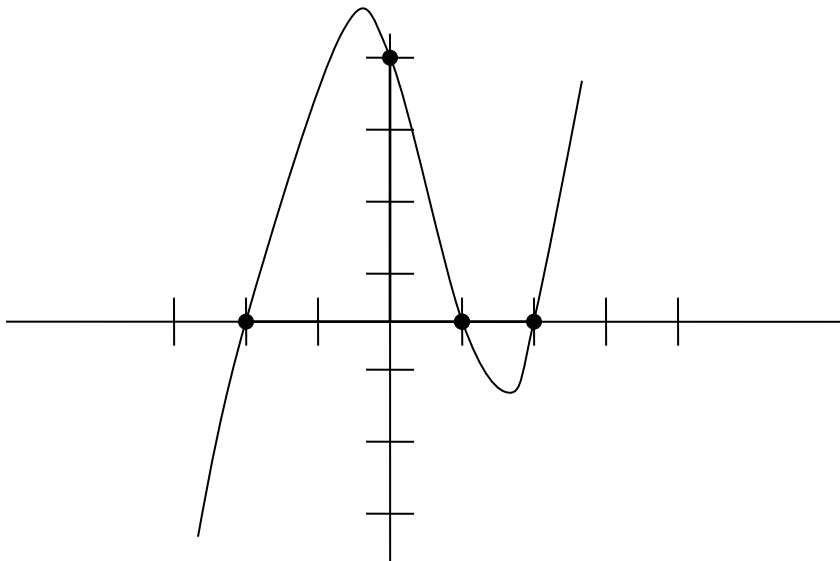
$$\begin{array}{r|rrrr} 1 & 1 & -1 & -4 & 4 \\ & & +1 & 0 & -4 \\ \hline & 1 & 0 & -4 & 0 \end{array} \rightarrow \text{so, } x = +1 \text{ is one root. The remaining polynomial is } x^2 - 4, \text{ which has two roots: } x = +2 \text{ and } x = -2.$$

Step 4:

- Therefore, $f(x) = (x - 1)(x - 2)(x + 2)$

Step 5:

- Find the y-intercept by determining $f(0) = (0)^3 - (0)^2 - 4(0) + 4 = 4$.
- Graph the function:



Step 6 (Other Special Tests):

- Descartes Rule of Signs**
 - 2 sign changes for $f(+x)$, 1 sign change for $f(-x)$ \rightarrow 2 positive roots, 1 negative root.