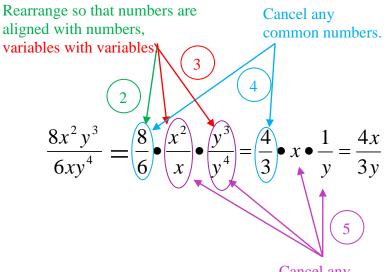# Simplifying Rational Expressions
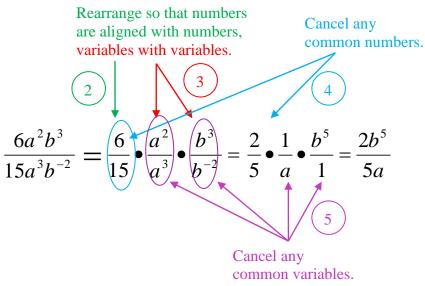
Look for: one fraction, only multiplication and division.

1. Factor if needed (often not needed).
2. Align numbers with numbers.
3. Align variables with like variables.
4. Cancel any common factors (numbers).
5. Cancel any common factors (variables).

Example 1 (no factoring):

Rearrange so that numbers are aligned with numbers, variables with variables.

Cancel any common numbers.

② ③ ④

$$\frac{8x^2y^3}{6xy^4} = \frac{8}{6} \bullet \frac{x^2}{x} \bullet \frac{y^3}{y^4} = \frac{4}{3} \bullet x \bullet \frac{1}{y} = \frac{4x}{3y}$$

⑤

Cancel any common variables.

Example 2 (no factoring):

Rearrange so that numbers are aligned with numbers, variables with variables.

Cancel any common numbers.

② ③ ④

$$\frac{6a^2b^3}{15a^3b^{-2}} = \frac{6}{15} \bullet \frac{a^2}{a^3} \bullet \frac{b^3}{b^{-2}} = \frac{2}{5} \bullet \frac{1}{a} \bullet \frac{b^5}{1} = \frac{2b^5}{5a}$$
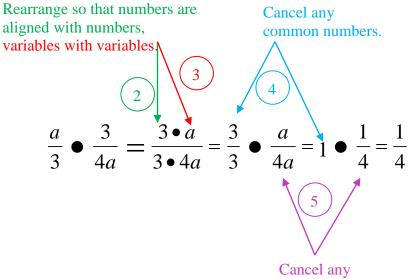
⑤

Cancel any common variables.

Multiplying Rational Expressions

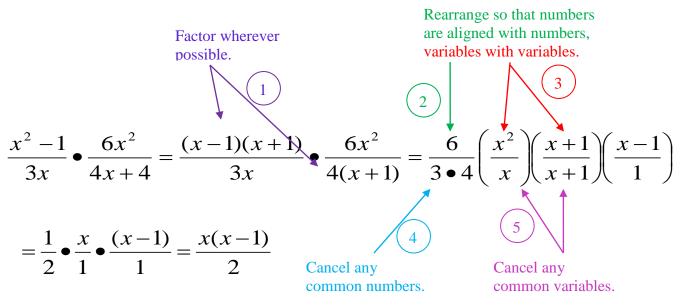*Look for: multiple fractions connected by a multiplication sign.*

1. Factor as much as possible.
2. Align numbers with numbers.
3. Align variables with like variables.
4. Cancel any common factors (numbers).
5. Cancel any common factors (variables).

Example (without factoring):

Rearrange so that numbers are aligned with numbers, variables with variables.

Cancel any common numbers.

(2) (3) (4)

$$\frac{a}{3} \bullet \frac{3}{4a} = \frac{3 \bullet a}{3 \bullet 4a} = \frac{3}{3} \bullet \frac{a}{4a} = 1 \bullet \frac{1}{4} = \frac{1}{4}$$

(5)

Cancel any common variables.

Example (with factoring):

Factor wherever possible.

Rearrange so that numbers are aligned with numbers, variables with variables.

(1) (2) (3)

$$\frac{x^2 - 1}{3x} \bullet \frac{6x^2}{4x + 4} = \frac{(x-1)(x+1)}{3x} \bullet \frac{6x^2}{4(x+1)} = \frac{6}{3 \bullet 4}\left(\frac{x^2}{x}\right)\left(\frac{x+1}{x+1}\right)\left(\frac{x-1}{1}\right)$$

(4)

Cancel any common numbers.

(5)

Cancel any common variables.

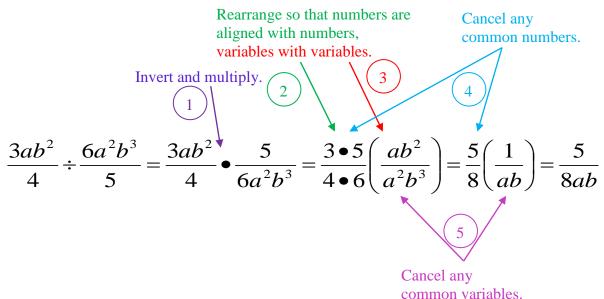$$= \frac{1}{2} \bullet \frac{x}{1} \bullet \frac{(x-1)}{1} = \frac{x(x-1)}{2}$$
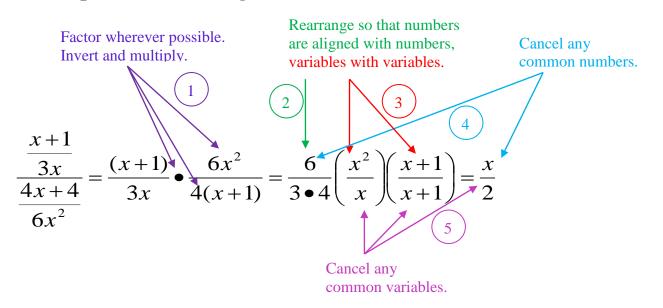
Dseriving Rational Expressions

Look for: multiple fractions connected by a divide sign or a fractional divide.

1. Factor as much as possible. Invert and multiply.
2. Align numbers with numbers.
3. Align variables with like variables.
4. Cancel any common factors (numbers).
5. Cancel any common factors (variables).

Example (without factoring and with divide sign):

Rearrange so that numbers are aligned with numbers, variables with variables.

Cancel any common numbers.

Invert and multiply.

$$\frac{3ab^2}{4} \div \frac{6a^2b^3}{5} = \frac{3ab^2}{4} \bullet \frac{5}{6a^2b^3} = \frac{3 \bullet 5}{4 \bullet 6}\left(\frac{ab^2}{a^2b^3}\right) = \frac{5}{8}\left(\frac{1}{ab}\right) = \frac{5}{8ab}$$

Cancel any common variables.

Example (with factoring and with fractional divide):

Factor wherever possible. Invert and multiply.

Rearrange so that numbers are aligned with numbers, variables with variables.

Cancel any common numbers.

$$\frac{\frac{x+1}{3x}}{\frac{4x+4}{6x^2}} = \frac{(x+1)}{3x} \bullet \frac{6x^2}{4(x+1)} = \frac{6}{3 \bullet 4}\left(\frac{x^2}{x}\right)\left(\frac{x+1}{x+1}\right) = \frac{x}{2}$$
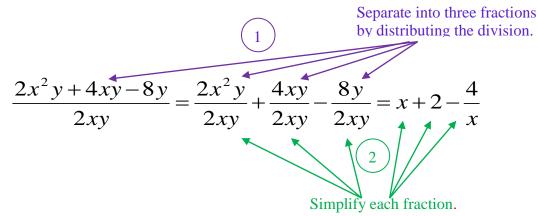
Cancel any common variables.

Dividing Rational Expressions (Distributed Division)

Look for: one fraction, multiple terms in denominator, single term in numerator.

1. Divide each term in the numerator by the term in the bottom.
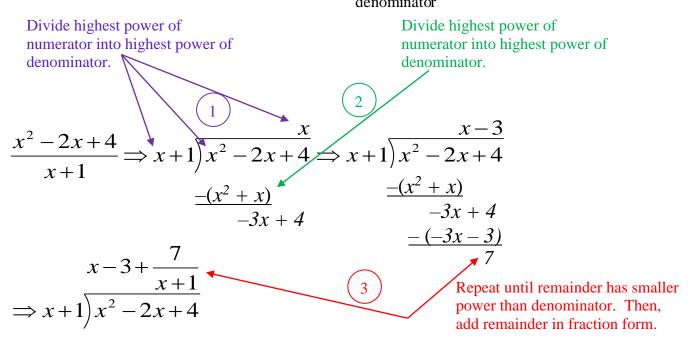2. Simplify each fraction as in "Simplifying Rational Expressions" above.

Example (distributed division):

Separate into three fractions by distributing the division.

①

$$\frac{2x^2y+4xy-8y}{2xy}=\frac{2x^2y}{2xy}+\frac{4xy}{2xy}-\frac{8y}{2xy}=x+2-\frac{4}{x}$$

②

Simplify each fraction.

Dividing Rational Expressions (Long Division)

Look for: one fraction, multiple terms in denominator, multiple terms in numerator.

1. Divide highest power of numerator by highest power of denominator.
2. Multiply and subtract.
3. Repeat until the remainder has a smaller power than the denominator. Then, add the remainder in fraction form ($\frac{\text{remainder}}{\text{denominator}}$).
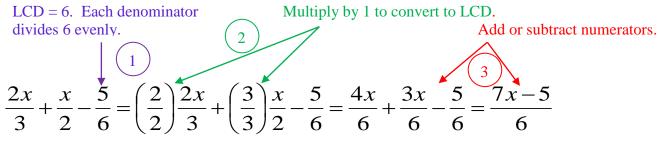
Divide highest power of numerator into highest power of denominator.

Divide highest power of numerator into highest power of denominator.

① ②

$$\frac{x^2-2x+4}{x+1}\Rightarrow x+1\overline{\smash{\big)}\,x^2-2x+4}$$

$$\begin{array}{r} x \\ x+1\overline{\smash{\big)}\,x^2-2x+4} \\ \underline{-(x^2+x)} \\ -3x+4 \end{array}$$

$$\begin{array}{r} x-3 \\ x+1\overline{\smash{\big)}\,x^2-2x+4} \\ \underline{-(x^2+x)} \\ -3x+4 \\ \underline{-(-3x-3)} \\ 7 \end{array}$$

③

$$x-3+\frac{7}{x+1}$$

$$\Rightarrow x+1\overline{\smash{\big)}\,x^2-2x+4}$$

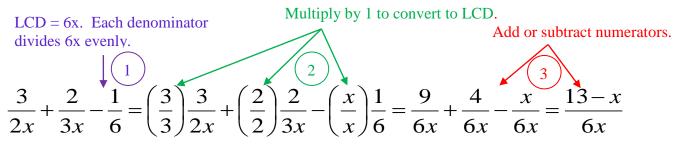Repeat until remainder has smaller power than denominator. Then, add remainder in fraction form.

Adding/Subtracting Rational Expressions

1. Find the least common denominator (LCD).
2. Multiple each fraction by 1 (in a form that converts each denominator into the LCD).
3. Add or subtract the numerators, keeping the LCD in the denominator.

Example (variables in numerator):

LCD = 6. Each denominator divides 6 evenly. ①

Multiply by 1 to convert to LCD. ②

Add or subtract numerators. ③

$$\frac{2x}{3}+\frac{x}{2}-\frac{5}{6}=\left(\frac{2}{2}\right)\frac{2x}{3}+\left(\frac{3}{3}\right)\frac{x}{2}-\frac{5}{6}=\frac{4x}{6}+\frac{3x}{6}-\frac{5}{6}=\frac{7x-5}{6}$$

Example (variables in denominator):

LCD = 6x. Each denominator divides 6x evenly. ①

Multiply by 1 to convert to LCD. ②

Add or subtract numerators. ③

$$\frac{3}{2x}+\frac{2}{3x}-\frac{1}{6}=\left(\frac{3}{3}\right)\frac{3}{2x}+\left(\frac{2}{2}\right)\frac{2}{3x}-\left(\frac{x}{x}\right)\frac{1}{6}=\frac{9}{6x}+\frac{4}{6x}-\frac{x}{6x}=\frac{13-x}{6x}$$

Solving Rational Expressions

1. Find the least common denominator (LCD).
2. Multiple each fraction by 1 (in a form that converts each denominator into the LCD).
3. Multiply by the LCD to cancel all denominators.
4. Solve for the variable.

Example (variables in denominator): $\dfrac{3}{2x}+\dfrac{5}{4x}=\dfrac{11}{8}$

LCD = 8x. Each denominator divides 8x evenly. ①

Multiply by 1 to convert to LCD. ②

Multiply by the LCD. ③

$$\frac{3}{2x}+\frac{5}{4x}=\frac{11}{8}\Rightarrow\left(\frac{4}{4}\right)\frac{3}{2x}+\left(\frac{2}{2}\right)\frac{5}{4x}=\left(\frac{x}{x}\right)\frac{11}{8}\Rightarrow\frac{12}{8x}+\frac{10}{8x}=\frac{11x}{8x}$$

Solve for the variable. ④

$$\Rightarrow 8x\left(\frac{12}{8x}+\frac{10}{8x}=\frac{11x}{8x}\right)\Rightarrow 12+10=11x\Rightarrow 22=11x\Rightarrow x=2$$