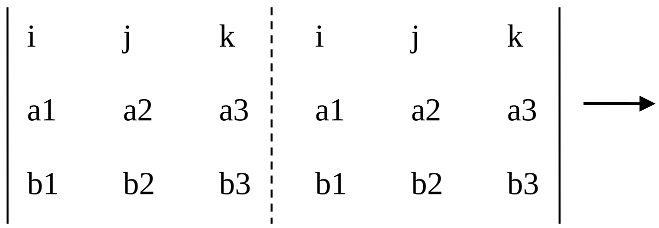
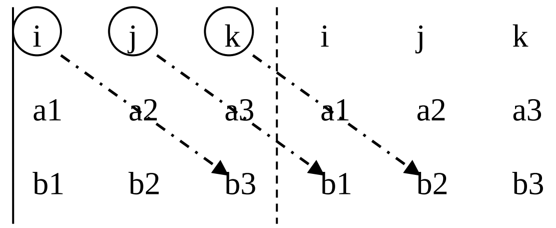
Vector Cross Product

Basics

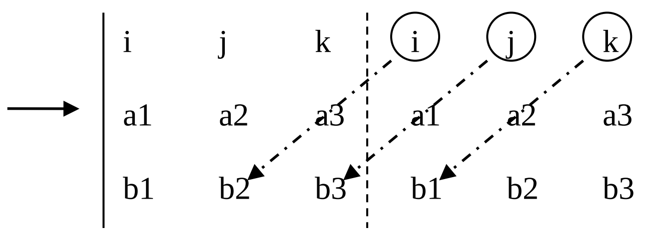
- First, the cross product **A x B** is a new vector that is perpendicular to both **A** and **B**.
- Given the two vectors **A** = <a1, a2, a3> and **B** = <b1, b2, b3>, the cross product **A x B** = <a2·b3 – a3·b2, a3·b1 – a1·b3, a1·b2 – a2·b1>.
- You can determine the cross product by the following approach which is used to compute the **determinant of the matrix.**



First, set up a matrix with the first row = **i j k**, the second row = vector **A**, and the third row = vector **B**. Then, duplicate the same three rows to the right.



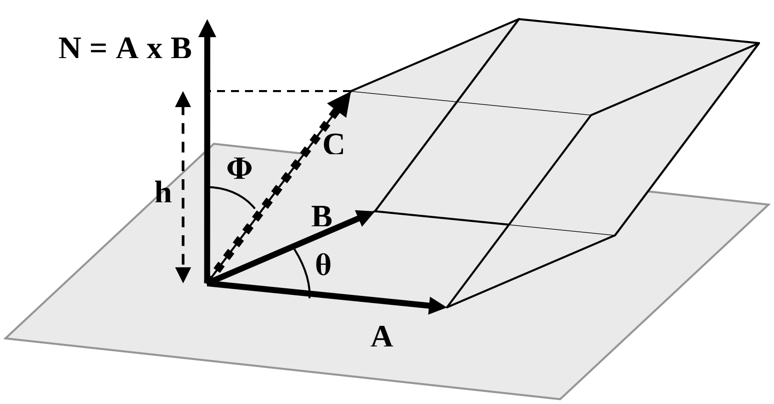
Get the positive terms by drawing diagonal lines from the the **i**, **j**, and **k** entries on the left. Multiply the entries to get (a2·b3)**i** + (a3·b1)**j** + (a1·b2)**k**.



Now, get the negative terms by drawing diagonal lines from the **i**, **j**, **k** entries on the right. Multiply the resultant entries to get –(a3·b2)**i** – (a1·b3)**j** – (a2·b1)**k.**

So, put it all together and you get **A x B** = (a2·b3 – a3·b2)**i** + (a3·b1 – a1·b3)**j** + (a1·b2 – a2·b1)**k**

- Note that the cross product **A x B** is defined = **n** | **A** | · | **B** | sin θ, where **n** is the unit vector perpendicular to both **A** and **B** and θ is the angle between **A** and **B**.
- If **A** and **B** are different vectors then the magnitude of **A x B** = | **A x B** | = | **A** | · | **B** | sin θ = the **area of the parallelogram** formed by the two vectors **A** and **B**.
- Note that if you have any vector **C** that is not in the same plane as **A** and **B**, then the three vectors form a **parallel-piped** (a box that is tilted at an angle Φ, the angle between **C** and **A x B**). The **volume of the parallel-piped** = (**A** x **B**) · **C**, which is the dot product of (**A x B)** with **C**. See the picture below. Note the parallelogram base formed by **A** and **B**.



A and **B** are in the plane. **C** is above the plane.