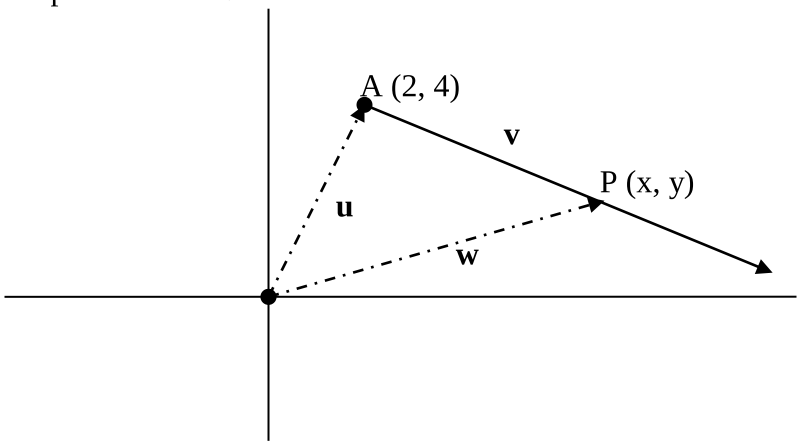
**Vector Representation of a Line**

Parametric & Vector Equation of a Line

- If you know a point on a line (A) and a vector (**v**) parallel to the line, you can find a vector equation and a parametric equation of the line.
- Here's a **2-D example** with A = (2, 4) and **v** = < 3, –1 >.
- First, identify the vector, **u**, from the origin to the point A. So, **u** = < 2, 4 >. See the picture below.




- Identify a vector, **w**, from the origin to a random point P (x, y) on the line. **w** = <x, y>.
- The vector **AP** is in the same direction as **v**, but has a different magnitude (length). That means there is a scalar t such that **AP** = t · **v**. This is the vector equation of the line.
- Also, **AP** = < x – 2, y – 4 > (just subtract the coordinates).
- So, that means < x – 2, y – 4 > = t · **v =** t · < 3, –1 > = <3t, –t >.
- That gives two parametric equations : x – 2 = 3t and y – 4 = –t (just match the corresponding components of the first and last vector above).
  - You can also write this as x = 2 + 3t and y = 4 – t.
  - Note that the parametric form has the coordinates of A (2, 4) as the first, constant values and the components of the vector <3, –1 > as the coefficients of t. **It always works this way.**
- If you want the Cartesian equation for the line, solve the two parametric equations for t and set them equal.
  - Solving the x equation gives $\frac{(x-2)}{3} = t$. Solving the y equation gives $\frac{y-4}{-1} = t$.
  - Setting them equal gives $\frac{(x-2)}{3} = \frac{y-4}{-1}$. Note again the A values on top and **v** values on the bottom. **It always works this way.**
  - As an alternative, in y = mx + b form, that gives $y = \frac{-1}{3}x + \frac{14}{3}$.
- The same approach works in **3-D**. **For example**, if you have a point A (1, 2, 3) and a vector <–1, 4, 8>, then **AP** = < x – 1, y – 2, z – 3 > = t · **v =** t · < –1, 4, 8 > = <–t, 4t, 8t >.
  - That gives three parametric equations: x – 1 = –t, y – 2 = 4t, and z – 3 = 8t.
  - Solve each equation for t and set them equal to get the Cartesian equation: $\frac{x-1}{-1} = \frac{y-2}{4} = \frac{z-3}{8}$. Again, see the A values on top and **v** values on the bottom.